Genetic Algorithms for EQ-algebras Automatic Generation

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Motivation

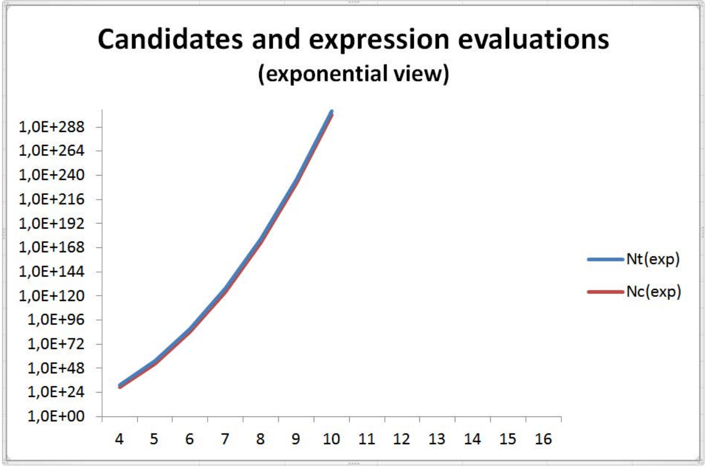
- Finite algebras generation with specific properties
- Task specification
  - n - number of algebra elements
  - Algebra operations declaration
  - Compulsory properties of operations
  - Optional properties of operations
  - Generate such algebra fulfilling requirements above
- Manual creation with help of properties automated check
- Brute force (combinatorial) approach
- More sophisticated methods?

Why not brute force?

- Example: n elements, k binary operations, l axioms (m elements dependence)
  - $N_c = (n)^{k*n*n}$ possible candidates
  - l axioms check - expression evaluations $N_{ev} = l * (n^m)$ for every candidate
  - total expression evaluations $N_t = N_c * N_{ev}$
  - expression means dozens of simple (CPU level) instructions
  - current common computer about $10^9 - 10^{10}$ instructions per second e.g. Intel Atom N270 - 3 GIPS, Intel Core i7 920 (Quad core) - 80 GIPS, SC IT4I (2015) cca 10^{15} IPS (FLOPS)...
- Fix $k = 3, l = 10, m = 3$
  - $\mathbf{n = 4}$, $\{0, a, b, 1\}$, $N_c \doteq 7.9 * 10^{28}$, $N_t \doteq \mathbf{5.1 * 10^{31}}$
  - $\mathbf{n = 5}$, $\{0, a, b, c, 1\}$, $N_c \doteq 2.6 * 10^{52}$, $N_t \doteq \mathbf{3.3 * 10^{55}}$
  - $\mathbf{n = 6}$, $\{0, a, b, c, d, 1\}$, $N_c \doteq 1.0 * 10^{84}$, $N_t \doteq \mathbf{2.4 * 10^{87}}$
  - $\mathbf{n = 7}$, $\{0, a, b, c, d, e, 1\}$, $N_c \doteq 1.6 * 10^{124}$, $N_t \doteq \mathbf{5.8 * 10^{127}}$
  - ...

Why not brute force?

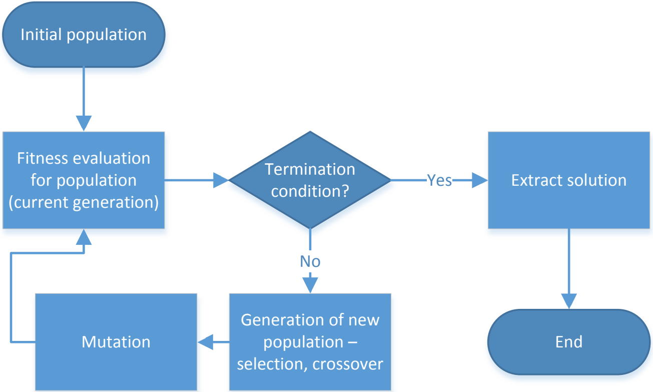
- "Hard" computing fails on superexponentiality of the problem (although many optimizations are possible, $o(n^n)$ remains)



Genetic algorithms (GA)

- Genetic algorithms - successful softcomputing method based on evolutionary principles
- User may select such parameters of GA to achieve "optimal" (not necessarily best) results in "reasonable" time in contrast to brute force
- Main characteristics:
  - Population member (candidate solution), its fitness function (evaluates suitability)
  - Population - set of members, starting population (random)
  - New generation created from previous by selection, crossover and mutation
  - Generate new populations until stop condition is fulfilled (fix number of iterations - populations, predefined member fitness being optimal etc.)

Genetic Algorithm Flowchart



Initial population
Fitness evaluation for population (current generation)
Termination condition?
Yes
Extract solution
No
Generation of new population – selection, crossover
Mutation
End

Population and Population Member (GA)

- Candidate solution p (Population Member / PM) represented by its properties (usually stored in "chromosomes" - bit array, integer array etc.)
- Fitness function of candidate solution f, $f(x) \in \langle 0, 1 \rangle$, x is PM - the keystone of time complexity of the task (possible parallelism)
- Population - fix or variable number of PM
    - Population member (candidate solution), its fitness function (evaluates suitability)
    - Population - sets of PMs, best PM, worst PM, median PM
    - Generation - sequence of populations called generations $G_0, \ldots, G_r$, where $G_i = \{p_{i,j} | i, j \in N\}$, i is generation index, j is PM index in population
    - Starting Generation G_0 is randomly (partially randomly) generated

Genetic operators (GA)

- Selection - simply into next generation or further processing
  - Elitist - usually best m PM from G_i is directly copied into G_{i+1}
  - Selection for crossover (SC) - some PMs from G_i are selected for generation of new children for G_{i+1},
  - SC should inhere probability of selection $prob_{SC}(p)$ for PM p non-decreasing with respect to fitness function: $f(p_1) \geq f(p_2) \Rightarrow prob_{SC}(p_1) \geq prob_{SC}(p_2)$
- Crossover - combination of several PMs to generate new PMs for next generation
  - Simple - two old PMs p_{old1}, p_{old2} generate two children, where first portion of chromosome is from p_{old1} and second from p_{old2} and contrary
  - Exponential - if we can distinguish several portions of chromosome we can generate more children than parents (every possible combination)

Genetic operators (GA)

- Mutation - randomly selected PMs from new generation are "altered"
  - Mutation rate - probability of selection PM for mutation
  - Point - single element of chromosome is altered
  - Interval - interval of chromosome elements are altered
  - Overall - whole chromosome is altered
- Termination - we have to end iterative application of operators to new generations
  - Best PM (average, median) - best PM in last population has fitness greater or equal to predefined value
  - Step - fixed number of steps (generations) is produced
  - Suitable PMs - predefined number of PMs with required fitness is generated
  - Time - time elapsed restriction to iteration
  - Peak - peak fitness is achieved and m next generations has worse fitness (or more sophisticated dependence on fitness)

Task - EQ-algebras generation

- EQ-algebras as truth value structure for EQ-logics
- Key operation - Fuzzy Equality
- 3 basic binary operations fulfilling several properties
  - Infimum $\wedge$
  - Multiplication $\otimes$
  - Fuzzy Equality $\sim$
- One possible additional unary operation
  - Delta Δ
- One derivable binary operation
  - Supremum (maximum) $\vee$
- Additional supporting (directly following) connectives
  - Implication $\rightarrow$
  - Negation $\neg$
  - LessOrEqual $\leq$

Task - EQ-algebras definition

EQ-algebra $\mathcal{E}$ - algebra of type (2, 2, 2, 0), $\mathcal{E} = \langle E, \wedge, \otimes, \sim, \mathbf{1} \rangle$

(E1) $\langle E, \wedge, \mathbf{1} \rangle$ is a commutative idempotent monoid (i.e. $\wedge$-semilattice with top element $\mathbf{1}$). We put $a \leq b$ iff $a \wedge b = a$, as usual.

(E2) $\langle E, \otimes, \mathbf{1} \rangle$ is a monoid and $\otimes$ is isotone w.r.t. $\leq$.

(E3) $a \sim a = \mathbf{1}$ (reflexivity axiom)

(E4) $((a \wedge b) \sim c) \otimes (d \sim a) \leq c \sim (d \wedge b)$ (substitution axiom)

(E5) $(a \sim b) \otimes (c \sim d) \leq (a \sim c) \sim (b \sim d)$ (congruence axiom)

(E6) $(a \wedge b \wedge c) \sim a \leq (a \wedge b) \sim a$ (monotonicity axiom)

(E7) $a \otimes b \leq a \sim b$ (boundedness axiom)

EQ-algebra - Additional operations

- Implication - $a \to b = (a \wedge b) \sim a$
- Negation - If $\mathcal{E}$ contains also the bottom element $\mathbf{0}$ then we put $\neg a = a \sim \mathbf{0}$ and call $\neg a$ a *negation* of $a \in E$.
- Maximum (supremum) - $\vee$ is derived from $\wedge$ preserving this condition: $(a \wedge b = a) \Rightarrow (a \vee b = b)$ (details in algorithms).
- Delta - EQ-algebra $\mathcal{E}$ extended by a unary additional operation $\varDelta : E \to E$ fulfilling the following axioms:

  (E$\varDelta$1) $\varDelta\mathbf{1} = \mathbf{1}$

  (E$\varDelta$2) $\varDelta a \leq \varDelta\varDelta a$

  (E$\varDelta$3) $\varDelta(a \sim b) \leq \varDelta a \sim \varDelta b$

  (E$\varDelta$4) $\varDelta(a \wedge b) = \varDelta a \wedge \varDelta b$

  (E$\varDelta$5) $\varDelta a = \varDelta a \otimes \varDelta a$

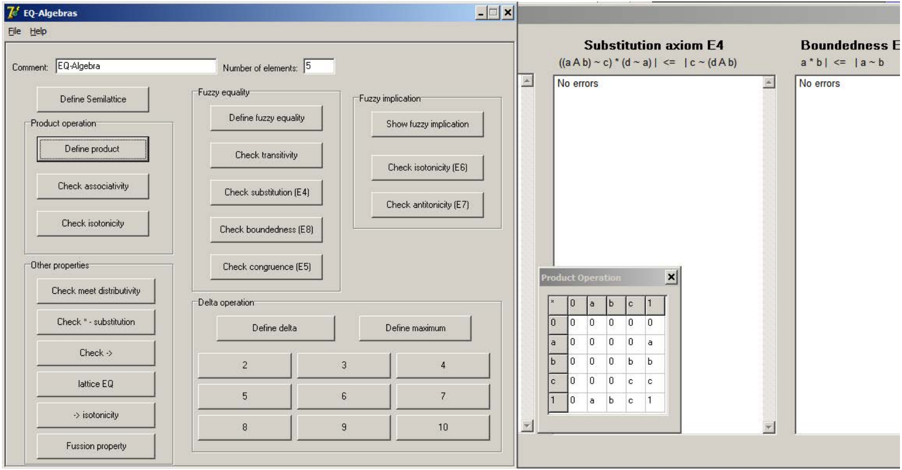
Special EQ-algebras

Let $\mathcal{E}$ be an EQ-algebra and $a, b, c, d \in E$. We say that $\mathcal{E}$ is:

1. *separated* if for all $a \in E$, $a \sim b = \mathbf{1}$ implies $a = b$.
2. *good* if $a \sim \mathbf{1} = a$.
3. *residuated* if for all $a, b, c \in E$, $(a \otimes b) \wedge c = a \otimes b$ iff $a \wedge ((b \wedge c) \sim b) = a$.
4. *involutive* (IEQ-algebra) if for all $a \in E$, $\neg\neg a = a$.
5. *prelinear* if for all $a, b \in E$, $\sup\{a \to b, b \to a\} = \mathbf{1}$.
6. *lattice EQ-algebra* (ℓEQ-algebra) if it is a lattice and $((a \vee b) \sim c) \otimes (d \sim a) \leq (d \vee b) \sim c$.
7. *linear* if for all $a, b \in E$ $((a \wedge b) = a)$ or $((a \wedge b) = b)$.

EQ-algebras - former support tool

Manual algebras design with automated axioms check (complicated for larger EQ-algebras)



Comment: EQ-Algebra

Number of elements: 5

Substitution axiom E4

((a A b) ~ c) * (d ~ a) | <= | c ~ (d A b)

No errors

Boundedness E

a * b | <= | a ~ b

No errors

*	0	a	b	c	1
0	0	0	0	0	0
a	0	0	0	0	a
b	0	0	0	b	b
c	0	0	0	c	c
1	0	a	b	c	1

Basic principles

- Object oriented model of EQ-algebras as GA Population Members
- GA Population (Generation) as *list* of PMs
- Fitness function based on relative fulfilment of mandatory and optional axioms
- EQ-algebras fulfilling additional criteria called Winners
- Winners are stored during GA process
- Very important is detection of previously generated (identical) candidates (removal)

PM and operations data structures

```
OperationTriple= array[1..3] of char;
OperationCouple= array[1..2] of char;

PEQAlgebra = ^TEQAlgebra;
TEQAlgebra = class
public
  NElements : integer;
  Elements : array[1..MaxNElements] of char;
  NSemilatticeArguments: integer; {number of different tuple
                                   in semilattice}
  SemiLattice : array [1..MaxNArguments] of OperationTriple;
  {only specific triples (x, y, x o y) in a triangle without
    of the operation square are stored}
...
```

Population data structure

```
PEQPopulation = ^TEQPopulation;
TEQPopulation = class(TList)
public
  parent, child : PEQPopulation;
  ElementsNo : integer;
  ...
  constructor Create();overload;
  procedure GenerateRandom(populationsize, elementsize:integ
  procedure CrossOver(item1, item2:PEQAlgebra; target:PEQPop
  procedure Mutate(prob:real);
  procedure RecomputeFit(win:PEQPopulation);
  procedure RemoveEqual();
...
```

GA algorithm detailed

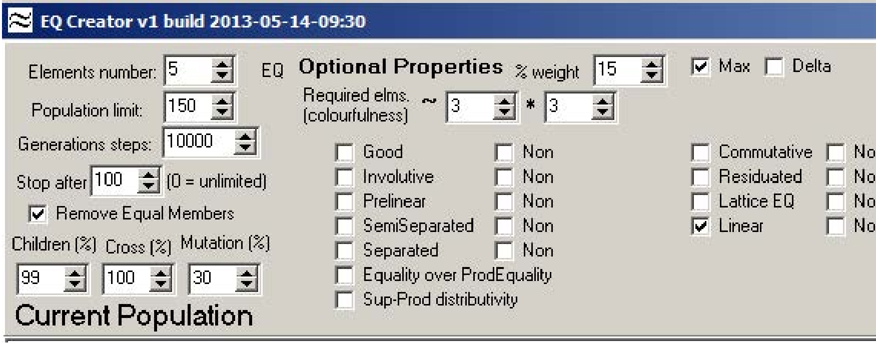
- Random (starting) population partially built to fulfil simple properties (e.g. infimum is commutative)
- Fitness evaluation - two phases:
  - Mandatory properties evaluation (e.g. boundedness axiom - $a \otimes b \leq a \sim b$)
  - Optional properties evaluation (e.g. goodness - $a \sim \mathbf{1} = a$)
- String representing a candidate algebra
- Removal of same candidates (based on the string representation)
- Sort of PMs in population through fitness
- Termination condition:
  - Fixed number of steps performed
  - Fixed number of EQ-algebras with required properties
  - Manual (user) termination

EQCreator application

- Algorithms implemented in the form of PC application EQCreator
- GUI based application for MS Windows 32-bit platform
- Former EQAlgebras tool written in Object Pascal language
- Minor usage of code - for backward compactibility (enables to load and save older eqa format
- Uses abstract types of Visual Component Library (TList)
- Main purpose:
    - Selection of various properties for candidate EQ-algebras
    - Evolution of algebras to attain EQ-algebras even with specific properties
    - Automated check of properties and generation
    - Saving of resulting optimal solutions in suitable form

EQCreator - basic functions

- Fundamental settings



EQ Creator v1 build 2013-05-14-09:30

Elements number: 5

Population limit: 150

Generations steps: 10000

Stop after 100 (0 = unlimited)

☑ Remove Equal Members

Children (%)	Cross (%)	Mutation (%)
99	100	30

Current Population

EQ **Optional Properties** % weight 15

☑ Max □ Delta

Required elms. (colourfulness) ~ 3 * 3

□ Good	□ Non	□ Commutative	□ No
□ Involutive	□ Non	□ Residuated	□ No
□ Prelinear	□ Non	□ Lattice EQ	□ No
□ SemiSeparated	□ Non	☑ Linear	□ No
□ Separated	□ Non		
□ Equality over ProdEquality			
□ Sup-Prod distributivity			

- Algebra elements number - support size (2 - 28)
- Population limit - max. number of algebras in population
- Generation steps - max. number of GA steps until one run stops (except stopped manually) (0 - unlimited)
- Stop after certain number of EQ-algebras found

EQCreator - Genetic algorithms settings

- Children ratio (0 - 100%) - crossover resulting new members relative count (how large portion of new population to be new children, others are old members copied from previous generation)
- Cross ratio (0 - 100%) - portion of BEST members to have possibility to crossover (it is not crossover probability!)
- Mutation ratio (0 - 100%) - probability for new population member to be mutated
- Crossover probability is set arbitrary (fixed) - in descending ordered (by fitness) population of the size N we set probability of member i $p_i = \frac{N-i}{\frac{N*(N+1)}{2}}$ for $i = 0, ..., N-1$, where $f(i) \geq f(i+1)$ (fitness for members)
  e.g. for 5 members: $p_0 = \frac{5}{15}, p_1 = \frac{4}{15}, ..., p_4 = \frac{1}{15}$

EQCreator - Optional settings

- weight of optional properties - relative weight of special EQ-algebras requirements (e.g. linear EQA, involutive EQA) - should be significantly less than for compulsory axioms (experimental best - 15%)
- notion of colourfulness - required number of distinct elements in variable positions for operator function values (some combinations are determined e.g. $a \wedge 0 = 0$ in every EQA)
- **colourfulness** assures non-trivial EQ-algebras to be generated e.g. for fuzzy equality when 3 of 5 required - at least 3 different elements occur as functional values in non-determined cases
- colourfulness experimentally needed for Product ($\otimes$) and Fuzzy Equality ($\sim$) - higher means computationally harder!

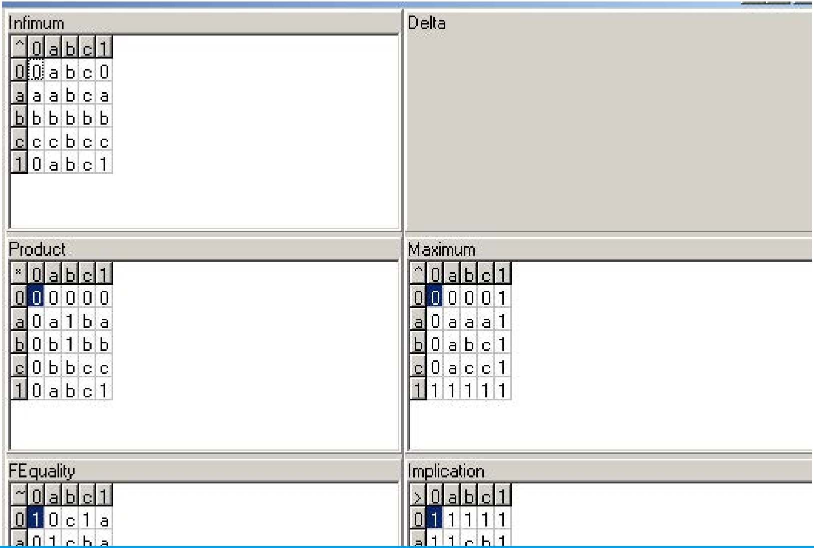
EQCreator - Optional settings

- Extension of EQ-algebras - **Max** and **Delta** operators - some **additional axioms** must hold for these operators!
- Special EQ-algebras holding (or not holding) additional axioms as optional selection:

  Good Commutative Involutive Residuated
  Prelinear Lattice Semiseparated Linear
  Separated etc.

- Important setting - **removal of equal** EQ-algebras from population!

EQCreator - Population members or Winners (EQA) Browsing



Infimum

^	0	a	b	c	1
0	0	a	b	c	0
a	a	a	b	c	a
b	b	b	b	b	b
c	c	c	b	c	c
1	0	a	b	c	1

Delta

Product

*	0	a	b	c	1
0	0	0	0	0	0
a	0	a	1	b	a
b	0	b	1	b	b
c	0	b	b	c	c
1	0	a	b	c	1

Maximum

^	0	a	b	c	1
0	0	0	0	0	1
a	0	a	a	a	1
b	0	a	b	c	1
c	0	a	c	c	1
1	1	1	1	1	1

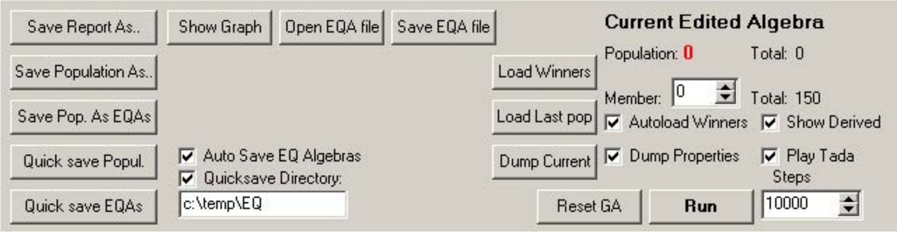
FEquality

~	0	a	b	c	1
0	1	0	c	1	a
a	0	1	c	b	a

Implication

>	0	a	b	c	1
0	1	1	1	1	1
a	1	1	c	b	1

EQCreator - Input and Output



Current Edited Algebra

Population: 0 Total: 0

Member: 0 Total: 150

☑ Autoload Winners ☑ Show Derived

☑ Dump Properties ☑ Play Tada

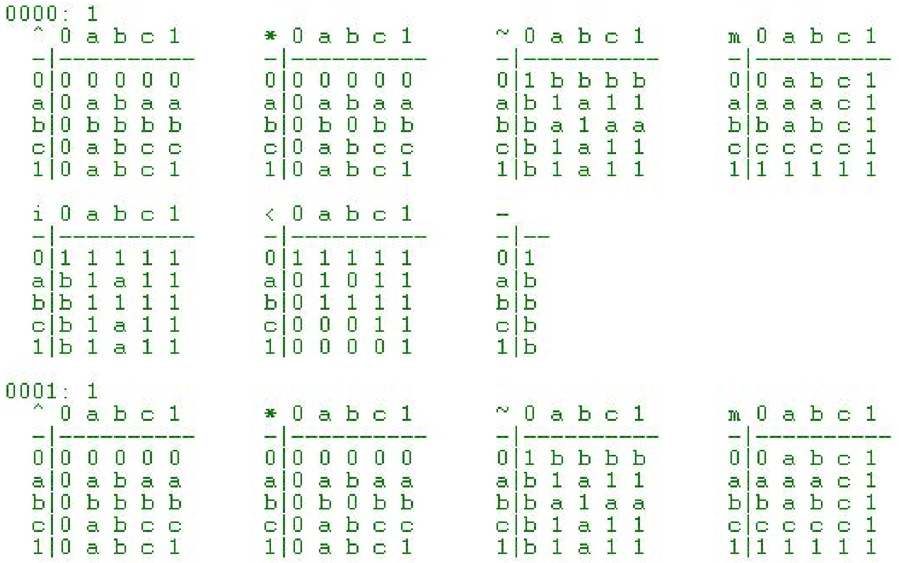
☑ Auto Save EQ Algebras

☑ Quicksave Directory: c:\temp\EQ

Steps 10000

- Open old EQA format (user can use formerly created algebras)
- Save both old EQA format of EQP - EQP is EQ-algebras (or general algebras) Population List
- EQP is in contrast to EQA readable text file with operator tables
- Number of GA steps could be limited

EQCreator - EQP output

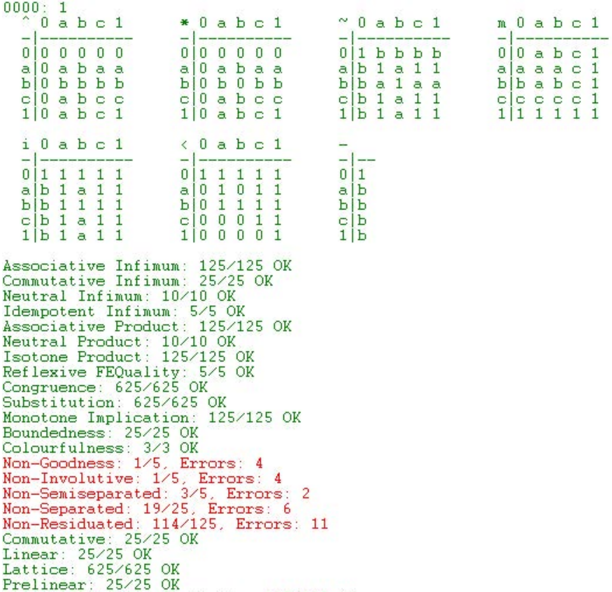


```
0000: 1
  ^ 0 a b c 1      * 0 a b c 1      ~ 0 a b c 1      m 0 a b c 1
  -|----------     -|----------     -|----------     -|----------
  0|0 0 0 0 0      0|0 0 0 0 0      0|1 b b b b      0|0 a b c 1
  a|0 a b a a      a|0 a b a a      a|b 1 a 1 1      a|a a a c 1
  b|0 b b b b      b|0 b 0 b b      b|b a 1 a a      b|b a b c 1
  c|0 a b c c      c|0 a b c c      c|b 1 a 1 1      c|c c c c 1
  1|0 a b c 1      1|0 a b c 1      1|b 1 a 1 1      1|1 1 1 1 1

  i 0 a b c 1      < 0 a b c 1      -
  -|----------     -|----------     -|--
  0|1 1 1 1 1      0|1 1 1 1 1      0|1
  a|b 1 a 1 1      a|0 1 0 1 1      a|b
  b|b 1 1 1 1      b|0 1 1 1 1      b|b
  c|b 1 a 1 1      c|0 0 0 1 1      c|b
  1|b 1 a 1 1      1|0 0 0 0 1      1|b

0001: 1
  ^ 0 a b c 1      * 0 a b c 1      ~ 0 a b c 1      m 0 a b c 1
  -|----------     -|----------     -|----------     -|----------
  0|0 0 0 0 0      0|0 0 0 0 0      0|1 b b b b      0|0 a b c 1
  a|0 a b a a      a|0 a b a a      a|b 1 a 1 1      a|a a a c 1
  b|0 b b b b      b|0 b 0 b b      b|b a 1 a a      b|b a b c 1
  c|0 a b c c      c|0 a b c c      c|b 1 a 1 1      c|c c c c 1
  1|0 a b c 1      1|0 a b c 1      1|b 1 a 1 1      1|1 1 1 1 1
```

EQCreator - EQP output with axiom fulfilment info

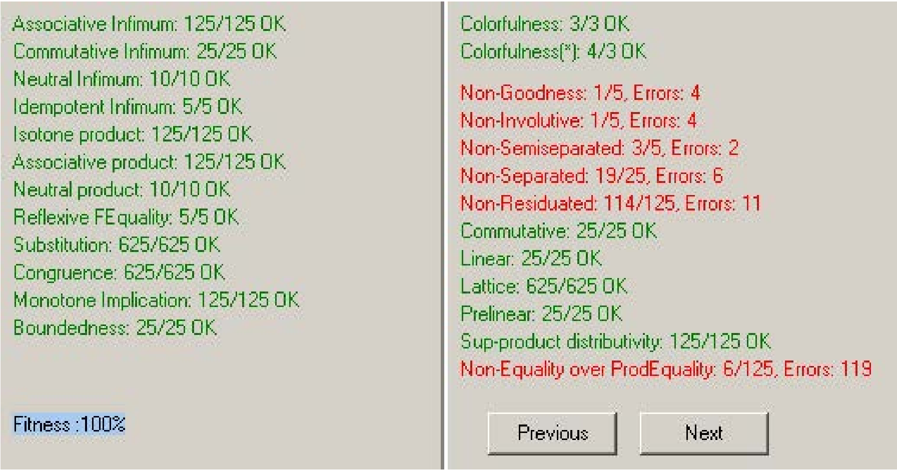


```
0000: 1
  ^ 0 a b c 1      * 0 a b c 1      ~ 0 a b c 1      m 0 a b c 1
  -|----------     -|----------     -|----------     -|----------
  0|0 0 0 0 0      0|0 0 0 0 0      0|1 b b b b      0|0 a b c 1
  a|0 a b a a      a|0 a b a a      a|b 1 a 1 1      a|a a a c 1
  b|0 b b b b      b|0 b 0 b b      b|b a 1 a a      b|b a b c 1
  c|0 a b c c      c|0 a b c c      c|b 1 a 1 1      c|c c c c 1
  1|0 a b c 1      1|0 a b c 1      1|b 1 a 1 1      1|1 1 1 1 1

  i 0 a b c 1      < 0 a b c 1      -
  -|----------     -|----------     -|--
  0|1 1 1 1 1      0|1 1 1 1 1      0|1
  a|b 1 a 1 1      a|0 1 0 1 1      a|b
  b|b 1 1 1 1      b|0 1 1 1 1      b|b
  c|b 1 a 1 1      c|0 0 0 1 1      c|b
  1|b 1 a 1 1      1|0 0 0 0 1      1|b

Associative Infimum: 125/125 OK
Commutative Infimum: 25/25 OK
Neutral Infimum: 10/10 OK
Idempotent Infimum: 5/5 OK
Associative Product: 125/125 OK
Neutral Product: 10/10 OK
Isotone Product: 125/125 OK
Reflexive FEQuality: 5/5 OK
Congruence: 625/625 OK
Substitution: 625/625 OK
Monotone Implication: 125/125 OK
Boundedness: 25/25 OK
Colourfulness: 3/3 OK
Non-Goodness: 1/5, Errors: 4
Non-Involutive: 1/5, Errors: 4
Non-Semiseparated: 3/5, Errors: 2
Non-Separated: 19/25, Errors: 6
Non-Residuated: 114/125, Errors: 11
Commutative: 25/25 OK
Linear: 25/25 OK
Lattice: 625/625 OK
Prelinear: 25/25 OK
```

EQCreator - Compulsory and Optional Axioms real-time view



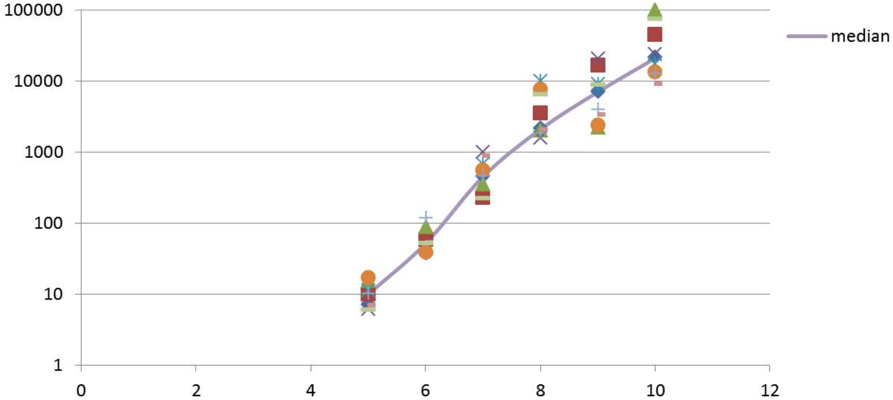
Associative Infimum: 125/125 OK
Commutative Infimum: 25/25 OK
Neutral Infimum: 10/10 OK
Idempotent Infimum: 5/5 OK
Isotone product: 125/125 OK
Associative product: 125/125 OK
Neutral product: 10/10 OK
Reflexive FEquality: 5/5 OK
Substitution: 625/625 OK
Congruence: 625/625 OK
Monotone Implication: 125/125 OK
Boundedness: 25/25 OK

Fitness :100%

Colorfulness: 3/3 OK
Colorfulness(*): 4/3 OK

Non-Goodness: 1/5, Errors: 4
Non-Involutive: 1/5, Errors: 4
Non-Semiseparated: 3/5, Errors: 2
Non-Separated: 19/25, Errors: 6
Non-Residuated: 114/125, Errors: 11
Commutative: 25/25 OK
Linear: 25/25 OK
Lattice: 625/625 OK
Prelinear: 25/25 OK
Sup-product distributivity: 125/125 OK
Non-Equality over ProdEquality: 6/125, Errors: 119

EQCreator - time efficiency



Tested on Pentium 4 - 2.8 GHz. In contrast to state space searching significant difference (no superexponentiality)

Conclusions

- Genetic algorithms made the task solvable in sensible time
- Specific GA properties are required:
    - Elitism must be used at least of minimal level (5% was acceptable - of course higher usage leads to worse convergence)
    - High mutation ratio must be set in contrast with traditional use of GA (best results with 20 - 30%)
    - Optional axioms and requirements need to have significantly less weight (experimentally 15% has best results)
    - Optional properties negatively affect convergence
    - Colourfulness was defined to prevent trivial solutions (evolution tends to most simple way of achieving results)
- EQ Creator - software for EQ-algebras only, but we suppose to bring fully general generator for algebras

Thank you for attention.