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НИИ Математико-информационных основ обучения

Препринт №21

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ON EXCEPTIONAL NIL OF INDEX 3 JORDAN ALGEBRAS

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Abstract. The exceptional nil of index 3 Jordan algebras are constructed in this paper.

1. **Introduction.** All algebras are considered over a field F of characteristic 0, so defining identities are linearized. Jordan multiplication will be denoted by point. We shall use right-handed bracketing in nonassociative words. We shall call a variety as a special if all its algebras have an associative envelope. Standard definitions and notations can be found in [1].

Let ***SpecN, N, SN*** are class of all special nil of index 3 Jordan algebras, variety of nil index 3 of Jordan algebras and variety generated by all special algebras from ***N***, respectively.

The problem of existing of exceptional nil of index 3 Jordan Algebras is well-known. Prof. Shestakov discussed the problem of speciality or nonspeciality of ***N*** with one of the authors of this paper as far back as 1977.

Variety ***N*** is a commutative analogy of ***Lie*** variety, i.e. it is determined by the commutative identity and the Jacobi identity.

$$J(x,y,z) = x\cdot y\cdot z + y\cdot z\cdot x + z\cdot x\cdot y = 0. \qquad (1)$$

Indeed, in a view of (1)

$$(x^2\cdot y)\cdot x = -x^3\cdot y - x^2\cdot(y\cdot x) = -x^2\cdot(y\cdot x) = 2y\cdot x\cdot x\cdot x = -x^2\cdot y\cdot x,$$

and

$$(x^2\cdot y)\cdot x = -x^2\cdot y\cdot x = 0.$$

Thus, any commutative algebra with Jacobi identity belongs to ***N***. The reverse is obvious.

Well-known [2] that ***Lie*** variety is a special one.

Note that if $J \in N$ and ***Ann*** $(J) = 0$ then J is a special algebra [1]. In particular, examples given by Zhelvakov and Shestakov are special algebras.

These facts were the initial point for a hypothesis about speciality of ***N***. Numerous attempts to prove this hypothesis have been failure. Using an interactive software package "ALBERT" [3] for a studying of identities in nonassociative algebra, authors managed to construct example of exceptional algebra from ***N***.

In this paper we shall prove that

$$SpecN \subsetneq SN \subsetneq N$$

We shall use the computer-derived basis and multiplication table of exceptional algebra from N. But all proofs in the current paper, in particularly, that $A \in N$ and $A \notin SN$ are complete and "hand-made" (i.e., without computer assistance).

2. **Example**. Let us consider the set of nonassociative words $E = \{e_1, ..., e_{44}\}$ with independent generators a, b, c. Let us truck down all elements of E denoting a length d(w) of a word w. Let us denote the types of word w as t(w) = (i, j, k), where i, j, k are a degree of word w by a, b, c correspondingly:

length 1:

$e_1 = a$, (1, 0, 0); $e_2 = b$, (0, 1, 0); $e_3 = c$, (0, 0, 1);

length 2:

$e_4 = c^2$, (0, 0, 2); $e_5 = bc$, (0, 1, 1); $e_6 = b^2$, (0, 2, 0);
$e_7 = ca$, (1, 0, 1); $e_8 = ba$, (1, 1, 0); $e_9 = a^2$, (2, 0, 0);

length 3:

$e_{10} = bcc$, (0, 1, 2); $e_{11} = b^2c$, (0, 2, 1); $e_{12} = acc$, (1, 0, 2);
$e_{13} = acb$, (1, 1, 1); $e_{14} = abc$, (1, 1, 1); $e_{15} = abb$, (1, 0, 2);
$e_{16} = a^2c$, (2, 0, 1); $e_{17} = a^2b$, (2, 1, 0);

length 4:

$e_{18} = b^2cc$, (0, 2, 2); $e_{19} = acbc$, (1, 1, 2); $e_{20} = abcc$, (1, 1, 2);
$e_{21} = abcb$, (1, 2, 1); $e_{22} = abbc$, (1, 2, 1); $e_{23} = a^2cc$, (2, 0, 2);
$e_{24} = a^2cb$, (2, 1, 1); $e_{25} = a^2bc$, (2, 1, 1); $e_{26} = a^2bb$, (2, 2, 0);

length 5:

$e_{27} = abcbc$, (1, 2, 2); $e_{28} = abbcc$, (1, 2, 2); $e_{29} = abbcb$, (1, 3, 1);
$e_{30} = a^2cbc$, (2, 1, 2); $e_{31} = a^2bcc$, (2, 1, 2); $e_{32} = a^2bcb$, (2, 2, 1);
$e_{33} = a^2bbc$, (2, 2, 1); $e_{34} = a^2bca$, (3, 1, 1);

length 6:

$e_{35} = abbcbc$, (1, 3, 2); $e_{36} = a^2bcbc$, (2, 2, 2); $e_{37} = a^2bbcc$, (2, 2, 2);
$e_{38} = a^2bbcb$, (2, 3, 1); $e_{39} = a^2bcac$, (3, 1, 2); $e_{40} = a^2bcab$, (3, 2, 1);

length 7:

$e_{41} = a^2bbcbc$, (2, 3, 2); $e_{42} = a^2bcabc$, (3, 2, 2); $e_{43} = a^2bcabb$, (3, 3, 1);

length 8:

$e_{44} = a^2bcabbc$, (3, 3, 2).

We convert the free F-module A over E into algebra by defining commutative multiplication on the basis E according to the following rules:

$e_1e_1 = e_9$,
$e_1e_2 = e_8$,
$e_1e_3 = e_7$,
$e_1e_4 = -2e_{12}$,
$e_1e_{13} = -\frac{1}{2}e_{25}$,
$e_1e_{14} = -\frac{1}{2}e_{24}$,
$e_1e_{15} = -\frac{1}{2}e_{26}$,
$e_1e_{18} = 2e_{27} + 2e_{28}$,
$e_1e_{28} = -\frac{1}{2}e_{36}$
$e_1e_{29} = \frac{1}{2}e_{38}$,
$e_1e_{31} = -e_{39}$,
$e_1e_{33} = -e_{40}$,

$e_1e_5 = -e_{13} - e_{14}$,
$e_1e_6 = -2e_{15}$,
$e_1e_7 = -\frac{1}{2}e_{16}$,
$e_1e_8 = -\frac{1}{2}e_{17}$,
$e_1e_{10} = e_{19}$,
$e_1e_{11} = -2e_{21}$,
$e_1e_{12} = -\frac{1}{2}e_{23}$,
$e_1e_{19} = -\frac{1}{2}e_{30}$,
$e_1e_{20} = \frac{1}{2}e_{30} + \frac{1}{2}e_{31}$,
$e_1e_{21} = -\frac{1}{2}e_{32}$,
$e_1e_{22} = \frac{1}{2}e_{32} + \frac{1}{2}e_{33}$,
$e_1e_{24} = -e_{34}$,
$e_1e_{25} = e_{34}$,
$e_1e_{27} = -\frac{1}{2}e_{37}$,
$e_1e_{35} = \frac{1}{2}e_{41}$,
$e_1e_{36} = -e_{42}$,
$e_1e_{37} = e_{42}$,
$e_1e_{38} = e_{43}$,
$e_1e_{41} = -\frac{1}{2}e_{44}$,

$e_2e_2 = e_6$,
$e_2e_3 = e_5$,
$e_2e_4 = -2e_{10}$,
$e_2e_5 = -\frac{1}{2}e_{11}$,
$e_2e_7 = e_{13}$,
$e_2e_8 = e_{15}$,
$e_2e_9 = e_{17}$,
$e_2e_{10} = -\frac{1}{2}e_{18}$,
$e_2e_{12} = -e_{19} - e_{20}$,
$e_2e_{13} = -e_{21} - e_{22}$,
$e_2e_{14} = e_{21}$,
$e_2e_{16} = e_{24}$,
$e_2e_{17} = e_{26}$,
$e_2e_{19} = e_{28}$,
$e_2e_{20} = -e_{27} - e_{28}$,
$e_2e_{21} = -e_{29}$,
$e_2e_{22} = e_{29}$,
$e_2e_{23} = -e_{30} - e_{31}$,
$e_2e_{24} = -e_{32} - e_{33}$,
$e_2e_{25} = e_{32}$,
$e_2e_{28} = -e_{35}$,
$e_2e_{30} = e_{37}$,
$e_2e_{31} = -e_{36} - e_{37}$,
$e_2e_{32} = -e_{38}$,
$e_2e_{33} = e_{38}$,
$e_2e_{34} = e_{40}$,
$e_2e_{37} = -e_{41}$,
$e_2e_{39} = e_{42}$,
$e_2e_{40} = e_{43}$,
$e_2e_{42} = -\frac{1}{2}e_{44}$,

$e_3e_3 = e_4$,
$e_3e_5 = e_{10}$,
$e_3e_6 = e_{11}$,
$e_3e_7 = e_{12}$,
$e_3e_8 = e_{14}$,
$e_3e_9 = e_{16}$,
$e_3e_{11} = e_{18}$,
$e_3e_{13} = e_{19}$,
$e_3e_{14} = e_{20}$,
$e_3e_{15} = e_{22}$,
$e_3e_{16} = e_{23}$,
$e_3e_{17} = e_{25}$,
$e_3e_{21} = e_{27}$,
$e_3e_{22} = e_{28}$,
$e_3e_{24} = e_{30}$,
$e_3e_{25} = e_{31}$,
$e_3e_{26} = e_{33}$,
$e_3e_{29} = e_{35}$,
$e_3e_{32} = e_{36}$,
$e_3e_{33} = e_{37}$,
$e_3e_{34} = e_{39}$,
$e_3e_{38} = e_{41}$,
$e_3e_{40} = e_{42}$,
$e_3e_{43} = e_{44}$,

$e_4e_6 = -2e_{18}$,
$e_4e_8 = -2e_{20}$,
$e_4e_9 = -2e_{23}$,
$e_4e_{15} = -2e_{28}$,
$e_4e_{17} = -2e_{31}$,
$e_4e_{26} = -2e_{37}$,

$e_5e_5 = e_{18}$,
$e_5e_7 = e_{20}$,
$e_5e_8 = -e_{21} - e_{22}$,
$e_5e_9 = -e_{24} - e_{25}$,
$e_5e_{13} = e_{27}$,
$e_5e_{14} = e_{28}$,
$e_5e_{15} = -e_{29}$,
$e_5e_{16} = e_{31}$,
$e_5e_{17} = -e_{32} - e_{33}$,
$e_5e_{21} = e_{35}$,
$e_5e_{24} = e_{36}$,
$e_5e_{25} = e_{37}$,
$e_5e_{26} = -e_{38}$,
$e_5e_{32} = e_{41}$,
$e_5e_{34} = -2e_{42}$,
$e_5e_{40} = -\frac{1}{2}e_{44}$,

$e_6e_7 = 2e_{21} + 2e_{22}$, $e_6e_9 = -2e_{26}$, $e_6e_{12} = -2e_{27}$, $e_6e_{14} = 2e_{29}$, $e_6e_{16} = 2e_{32} + 2e_{33}$,
$e_6e_{19} = 2e_{35}$, $e_6e_{20} = -2e_{35}$, $e_6e_{23} = -2e_{36}$, $e_6e_{25} = 2e_{38}$, $e_6e_{30} = 2e_{41}$,
$e_6e_{31} = -2e_{41}$, $e_6e_{34} = -2e_{43}$, $e_6e_{39} = e_{44}$,

$e_7e_7 = e_{23}$, $e_7e_8 = \frac{1}{2}(e_{24} + e_{25})$, $e_7e_{11} = -2e_{28}$, $e_7e_{13} = \frac{1}{2}(e_{30} + e_{31})$, $e_7e_{14} = -\frac{1}{2}e_{31}$,
$e_7e_{15} = -\frac{1}{2}e_{32}$, $e_7e_{17} = -e_{34}$, $e_7e_{21} = \frac{1}{2}(e_{36} + e_{37})$, $e_7e_{22} = -\frac{1}{2}e_{37}$, $e_7e_{24} = e_{39}$,
$e_7e_{26} = e_{40}$, $e_7e_{29} = -e_{41}$, $e_7e_{32} = e_{42}$, $e_7e_{38} = -\frac{1}{2}e_{44}$,

$e_8e_8 = e_{26}$, $e_8e_{10} = e_{27}$, $e_8e_{11} = -2e_{29}$, $e_8e_{12} = -\frac{1}{2}e_{30}$, $e_8e_{13} = \frac{1}{2}(e_{32} + e_{33})$, $e_8e_{14} = -\frac{1}{2}e_{33}$, $e_8e_{16} = e_{34}$,
$e_8e_{18} = 2e_{35}$, $e_8e_{19} = \frac{1}{2}(e_{36} + e_{37})$, $e_8e_{20} = -\frac{1}{2}e_{37}$, $e_8e_{22} = -\frac{1}{2}e_{38}$, $e_8e_{23} = -e_{39}$, $e_8e_{25} = -e_{40}$, $e_8e_{27} = -\frac{1}{2}e_{41}$,
$e_8e_{28} = \frac{1}{2}e_{41}$, $e_8e_{30} = -e_{42}$, $e_8e_{31} = e_{42}$, $e_8e_{32} = e_{43}$, $e_8e_{36} = -\frac{1}{2}e_{44}$.

$e_9e_{10} = e_{30}$, $e_9e_{11} = -2e_{32}$, $e_9e_{13} = e_{34}$, $e_9e_{14} = -e_{34}$.
$e_9e_{18} = 2(e_{36} + e_{37})$, $e_9e_{20} = e_{39}$, $e_9e_{22} = e_{40}$, $e_9e_{27} = e_{42}$.
$e_9e_{28} = -e_{42}$, $e_9e_{29} = -e_{43}$, $e_9e_{35} = \frac{1}{2}e_{44}$.

$e_{10}e_{15} = e_{35}$, $e_{10}e_{17} = e_{36}$, $e_{10}e_{26} = e_{41}$.

$e_{11}e_{13} = -2e_{35}$, $e_{11}e_{16} = -2e_{37}$, $e_{11}e_{17} = -2e_{38}$, $e_{11}e_{24} = -2e_{41}$, $e_{11}e_{34} = e_{44}$.

$e_{12}e_{15} = \frac{1}{2}(e_{36} + e_{37})$, $e_{12}e_{17} = e_{39}$, $e_{12}e_{26} = -e_{42}$.

$e_{13}e_{13} = e_{36}$, $e_{13}e_{14} = -(e_{36} + e_{37})$, $e_{13}e_{15} = -\frac{1}{2}e_{38}$,
$e_{13}e_{16} = -e_{39}$, $e_{13}e_{21} = -\frac{1}{2}e_{41}$, $e_{13}e_{22} = \frac{1}{2}e_{41}$,
$e_{13}e_{25} = -e_{42}$, $e_{13}e_{26} = -e_{43}$, $e_{13}e_{33} = \frac{1}{2}e_{44}$,

$e_{14}e_{14} = e_{37}$, $e_{14}e_{15} = \frac{1}{2}e_{38}$, $e_{14}e_{17} = e_{40}$, $e_{14}e_{21} = \frac{1}{2}e_{41}$, $e_{14}e_{24} = e_{42}$, $e_{14}e_{32} = -\frac{1}{2}e_{44}$,

$e_{15}e_{16} = -e_{40}$, $e_{15}e_{20} = -\frac{1}{2}e_{41}$, $e_{15}e_{23} = e_{42}$, $e_{15}e_{24} = e_{43}$, $e_{15}e_{30} = -\frac{1}{2}e_{44}$,

$e_{16}e_{21} = -e_{42}$, $e_{16}e_{29} = \frac{1}{2}e_{44}$,

$e_{17}e_{18} = 2e_{41}$, $e_{17}e_{19} = e_{42}$, $e_{17}e_{20} = -e_{42}$, $e_{17}e_{21} = -e_{43}$, $e_{17}e_{27} = \frac{1}{2}e_{44}$,

$e_{19}e_{26} = \frac{1}{2}e_{44}$, $e_{21}e_{25} = \frac{1}{2}e_{44}$, $e_{22}e_{24} = -\frac{1}{2}e_{44}$.

All the rest products of basis elements are equal to zero.

Let us point out simple conclusions of above multiplication table:
- algebra A is generated by elements a, b, c and nilpotent-of-index 9.
- if $e_ie_j \neq 0$ than

$$d(e_ie_j) = d(e_i) + d(e_j),$$
$$d_x(e_ie_j) = d_x(e_i) + d_x(e_j),$$

where d_x is degree in x and $x \in \{a, b, c\}$.
Hence, we have a correct definitions of the length, homogeneity, and degree of the homogenous element in generator for algebra A.
Let us consider a mapping $\varphi(a) = b$, $\varphi(b) = a$, $\varphi(c) = c$ and extend it to endomorphism F – module A, determined on the basis words e_i by changing places a and b and being c on the same place.

For instance:

$\varphi(e_{35}) = \varphi(abbcbc) = b\ a\ a\ c\ a\ c = e_2\ e_1\ e_1\ e_3\ e_1\ e_3 = -\frac{1}{2}e_{39}$.

From the multiplication table we can find action of φ on the basis:

length 1.	$\varphi(e_1) = e_2$,	$\varphi(e_2) = e_1$,	$\varphi(e_3) = e_3$.	
length 2.	$\varphi(e_4) = e_4$,	$\varphi(e_5) = e_7$,	$\varphi(e_6) = e_9$,	$\varphi(e_7) = e_5$,
	$\varphi(e_8) = e_8$,	$\varphi(e_9) = e_6$.		
length 3.	$\varphi(e_{10}) = e_{12}$,	$\varphi(e_{11}) = e_{16}$,	$\varphi(e_{12}) = e_{10}$,	$\varphi(e_{13}) = -e_{13} - e_{14}$,
	$\varphi(e_{14}) = e_{14}$,	$\varphi(e_{15}) = -\frac{1}{2}e_{17}$,	$\varphi(e_{16}) = e_{11}$,	$\varphi(e_{17}) = -2e_{15}$.
length 4.	$\varphi(e_{18}) = e_{23}$,	$\varphi(e_{19}) = -e_{19} - e_{20}$,	$\varphi(e_{20}) = e_{20}$,	$\varphi(e_{21}) = -\frac{1}{2}e_{24}$,
	$\varphi(e_{22}) = -\frac{1}{2}e_{25}$,	$\varphi(e_{23}) = e_{18}$,	$(e_{24}) = -2e_{21}$,	$\varphi(e_{25}) = -2e_{22}$,
	$\varphi(e_{26}) = e_{26}$.			
length 5.	$\varphi(e_{27}) = -\frac{1}{2}e_{30}$,	$\varphi(e_{28}) = -\frac{1}{2}e_{31}$,	$\varphi(e_{29}) = -\frac{1}{2}e_{34}$,	$\varphi(e_{30}) = -2e_{27}$,
	$\varphi(e_{31}) = -2e_{28}$,	$\varphi(e_{32}) = -e_{32} - e_{33}$,	$\varphi(e_{33}) = e_{33}$,	$\varphi(e_{34}) = -2e_{29}$.
length 6.	$\varphi(e_{35}) = -\frac{1}{2}e_{39}$,	$\varphi(e_{36}) = -e_{36} - e_{37}$,	$\varphi(e_{37}) = e_{37}$,	$\varphi(e_{38}) = -e_{40}$,
	$\varphi(e_{39}) = -2e_{35}$,	$\varphi(e_{40}) = -e_{38}$.		
length 7.	$\varphi(e_{41}) = -e_{42}$,	$\varphi(e_{42}) = -e_{41}$,	$\varphi(e_{43}) = -e_{43}$,	
length 8.	$\varphi(e_{44}) = -e_{44}$.			

We denote by A_i – F submodules of A which are generated by the homogeneous elements of the length i, $i = 1, \ldots, 8$.

Lemma 1. φ is automorphism of F-module of A ; A_i, $i = 1,\ldots,8$.

Proof. Prove at first that φ is automorphism F–module A.

Let $x = \sum_{i=1}^{44} \alpha_i e_i$, $\alpha_i \in F$. Then according to the definition φ we obtain

$$\varphi(x) = \sum_{i=1}^{44} \alpha'_i e_i,$$

where

$$
\begin{aligned}
&1.\ \alpha'_1 = \alpha_2,\ \alpha'_2 = \alpha_1,\ \alpha'_3 = \alpha_3.\\
&2.\ \alpha'_4 = \alpha_4,\ \alpha'_5 = \alpha_7,\ \alpha'_6 = \alpha_9,\ \alpha'_7 = \alpha_5,\ \alpha'_8 = \alpha_8,\ \alpha'_9 = \alpha_6.\\
&3.\ \alpha'_{10} = \alpha_{12},\ \alpha'_{11} = \alpha_{16},\ \alpha'_{12} = \alpha_{10},\ \alpha'_{13} = -\alpha_{13},\ \alpha'_{14} = -\alpha_{13} + \alpha_{14},\\
&\quad \alpha'_{15} = -2\alpha_{17},\ \alpha'_{16} = \alpha_{11},\ \alpha'_{17} = \frac{1}{2}\alpha_{15}.\\
&4.\ \alpha'_{18} = \alpha_{23},\ \alpha'_{19} = -\alpha_{19},\ \alpha'_{20} = -\alpha_{19} + \alpha_{20},\ \alpha'_{21} = -2\alpha_{24},\\
&\quad \alpha'_{22} = -2\alpha_{25},\ \alpha'_{23} = \alpha_{18},\ \alpha'_{24} = -\frac{1}{2}\alpha_{21},\ \alpha'_{25} = -\frac{1}{2}\alpha_{22},\ \alpha'_{26} = \alpha_{26}.\\
&5.\ \alpha'_{27} = -2\alpha_{30},\ \alpha'_{28} = -2\alpha_{31},\ \alpha'_{29} = -2\alpha_{34},\ \alpha'_{30} = -\frac{1}{2}\alpha_{27},\\
&\quad \alpha'_{31} = -\frac{1}{2}\alpha_{28},\ \alpha'_{32} = -\alpha_{32},\ \alpha'_{33} = -\alpha_{32} + \alpha_{33},\ \alpha'_{34} = -\frac{1}{2}\alpha_{29}.\\
&6.\ \alpha'_{35} = -2\alpha_{39},\ \alpha'_{36} = -\alpha_{36},\ \alpha'_{37} = -\alpha_{36} + \alpha_{37},\ \alpha'_{38} = -\alpha_{40},\\
&\quad \alpha'_{39} = -\frac{1}{2}\alpha_{35},\ \alpha'_{40} = -\alpha_{38}.\\
&7.\ \alpha'_{41} = -\alpha_{42},\ \alpha'_{42} = -\alpha_{41},\ \alpha'_{43} = -\alpha_{43}.\\
&8.\ \alpha'_{44} = -\alpha_{44}.
\end{aligned}
\tag{2}
$$

From (2) we obtain

$$x \in Ker\ \varphi \quad \Rightarrow \quad \alpha_i = 0 \quad \text{for all indices } i.$$

Consequently, $Ker\ \varphi = 0$, i.e. φ is automorphism. Note that φ preserves the length and, hence, φ is automorphism for A_i also. This proves the lemma.

Let

$$x = \sum_{i=1}^{44} \alpha_i e_i,\quad y = \sum_{i=1}^{44} \beta_i e_i,\quad x\,y = \sum_{i=1}^{44} \delta_i e_i,$$

where α_i, β_i, $\delta_i \in F$. Let us find δ_i from the multiplication table for algebra A. Let us introduce the following symbol:

$$[i, j] = \begin{cases} \alpha_i\beta_j + \alpha_j\beta_i, & i \neq j,\\ \alpha_i\beta_i, & i = j. \end{cases}$$

Note that $[i, j] = [j, i]$ for all i, j.

We obtain:

$$\delta_1 = \delta_2 = \delta_3 = 0,$$

$$\delta_4 = [3, 3], \delta_5 = [2, 3], \delta_6 = [2, 2], \delta_7 = [1, 3], \delta_8 = [1, 2], \delta_9 = [1, 1],$$

$$\delta_{10} = -2\,[2, 4] + [3, 5], \delta_{11} = -\frac{1}{2}\,[2, 5] + [3, 6], \delta_{12} = -2\,[1, 4] + [3, 7],$$

$$\delta_{13} = -[1, 5] + [2, 7], \delta_{14} = -[1, 5] + [3, 8], \delta_{15} = -2\,[1, 6] + [2, 8],$$

$$\delta_{16} = -\frac{1}{2}\,[1, 7] + [3, 9], \delta_{17} = -\frac{1}{2}\,[1, 8] + [2, 9],$$

$$\delta_{18} = -\frac{1}{2}\,[2, 10] + [3, 11] - 2\,[4, 6] + [5, 5],$$

$$\delta_{19} = [1, 10] - [2, 12] + [3, 13], \qquad (3)$$

$$\delta_{20} = -[2, 12] + [3, 14] - 2\,[4, 8] + [5, 7],$$

$$\delta_{21} = -2\,[1, 11] - [2, 13] + [2, 14] - [5, 8] + 2\,[6, 7],$$

$$\delta_{22} = -[2, 13] + [3, 15] - [5, 8] + 2\,[6, 7],$$

$$\delta_{23} = -\frac{1}{2}\,[1, 12] + [3, 16] - 2\,[4, 9] + [7, 7],$$

$$\delta_{24} = -\frac{1}{2}\,[1, 14] + [2, 16] - [5, 9] + \frac{1}{2}\,[7, 8],$$

$$\delta_{25} = -\frac{1}{2}\,[1, 13] + [3, 17] - [5, 9] + \frac{1}{2}\,[7, 8],$$

$$\delta_{26} = -\frac{1}{2}\,[1, 15] + [2, 17] - 2\,[6, 9] + [8, 8],$$

$$\delta_{27} = 2\,[1, 18] - [2, 20] + [3, 21] + [5, 13] - 2\,[6, 12] + [8, 10],$$

$$\delta_{28} = 2\,[1, 18] + [2, 19] - [2, 20] + [3, 22] - 2\,[4, 15] + [5, 14] - 2\,[7, 11],$$

$$\delta_{29} = -[2, 21] + [2, 22] - [5, 15] + 2\,[6, 14] - 2\,[8, 11],$$

$$\delta_{30} = -\frac{1}{2}\,[1, 19] + \frac{1}{2}\,[1, 20] - [2, 23] + [3, 24] + \frac{1}{2}\,[7, 13] - \frac{1}{2}\,[8, 12] + [9, 10],$$

$$\delta_{31} = \frac{1}{2}\,[1, 20] - [2, 23] + [3, 25] - 2\,[4, 17] + [5, 16] + \frac{1}{2}\,[7, 13] - \frac{1}{2}\,[7, 14],$$

$$\delta_{32} = -\frac{1}{2}\,[1, 21] + \frac{1}{2}\,[1, 22] - [2, 24] + [2, 25] - [5, 17] + 2\,[6, 16] - \frac{1}{2}\,[7, 15] + \frac{1}{2}\,[8, 13] - 2\,[9, 11],$$

$$\delta_{33} = \frac{1}{2}\,[1, 22] - [2, 24] + [3, 26] - [5, 17] + 2\,[6, 16] + \frac{1}{2}\,[8, 13] - \frac{1}{2}\,[8, 14],$$

$$\delta_{34} = -[1, 24] + [1, 25] - [7, 17] + [8, 16] + [9, 13] - [9, 14],$$

$$\delta_{35} = -[2, 28] + [3, 29] + [5, 21] + 2\,[6, 19] - 2\,[6, 20] + 2\,[8, 18] + [10, 15] - 2\,[11, 13],$$

$$\delta_{36} = -\frac{1}{2}\,[1, 28] - [2, 31] + [3, 32] + [5, 24] - 2\,[6, 23] + \frac{1}{2}\,[7, 21] + \frac{1}{2}\,[8, 19] + 2\,[9, 18] + [10, 17] + \frac{1}{2}\,[12, 15] + [13, 13] - [13, 14],$$

$$\delta_{37} = \frac{1}{2}\,[1, 27] + [2, 30] - [2, 31] + [3, 33] - 2\,[4, 26] + [5, 25] + \frac{1}{2}\,[7, 21] - \frac{1}{2}\,[7, 22] + \frac{1}{2} + [8, 19] - \frac{1}{2}\,[8, 20] + 2\,[9, 18] - 2\,[11, 16] + \frac{1}{2}\,[12, 15] - [13, 14] + [14, 14],$$

$$\delta_{38} = \frac{1}{2}\,[1, 29] - [2, 32] + [2, 33] - [5, 26] + 2\,[6, 25] - \frac{1}{2}\,[8, 22] - 2\,[11, 7] - \frac{1}{2}\,[13, 15] + \frac{1}{2}\,[14, 15],$$

$$\delta_{39} = -[1, 31] + [3, 34] + [7, 24] - [8, 23] + [9, 20] + [12, 17] - [13, 16],$$

$$\delta_{40} = -[1, 33] + [2, 34] + [7, 26] - [8, 25] + [9, 22] + [14, 17] - [15, 16],$$

$$\delta_{41} = \frac{1}{2}\,[1, 35] - [2, 37] + [3, 38] + [5, 32] + 2\,[6, 30] - 2\,[6, 31] - [7, 29] - \frac{1}{2}\,[8, 27] + \frac{1}{2}\,[8, 28] + [10, 26] - 2\,[11, 24] - \frac{1}{2}\,[13, 21] + \frac{1}{2}\,[13, 22] + \frac{1}{2}\,[14, 21] - \frac{1}{2}\,[15, 20] + 2\,[17, 18],$$

$$\delta_{42} = [1, 37] - [1, 36] + [2, 39] + [3, 40] - 2\,[5, 34] + [7, 32] - [8, 30] + [8, 31] + [9, 27] - [9, 28] - [12, 6] - [13, 25] + 14, 24] + [15, 23] - [16, 21] + [17, 19] - [17, 20],$$

$$\delta_{43} = [1, 38] + [2, 40] - 2\,[6, 34] + [8, 32] - [9, 29] - [13, 26] + [15, 24] - [17, 21],$$

$$\delta_{44} = -\frac{1}{2}\,[1, 41] - \frac{1}{2}\,[2, 42] + [3, 43] - \frac{1}{2}\,[5, 40] + [6, 39] - \frac{1}{2}\,[7, 38] - \frac{1}{2}\,[8, 36] + \frac{1}{2}\,[9, 35] + [11, 34] + \frac{1}{2}\,[13, 33] - \frac{1}{2}\,[14, 32] - \frac{1}{2}\,[15, 30] + \frac{1}{2}\,[16, 29] + \frac{1}{2}\,[17, 27] + \frac{1}{2}\,[19, 26] + \frac{1}{2}\,[21, 25] - \frac{1}{2}\,[22, 24].$$

Let

$$\varphi\,(x\,y) = \sum_{i=1}^{44} \delta'_i\, e_i, \quad \varphi\,(x) = \sum_{i=1}^{44} \alpha'_i\, e_i, \quad \varphi\,(y) = \sum_{i=1}^{44} \beta'_i\, e_i,$$

where δ'_i, α'_i, β'_i are calculated from formulae (2).

Let us introduce symbols

$$[i', j'] = \begin{cases} \alpha'_i\beta'_j + \alpha'_j\beta'_i, & i \neq j, \\ \alpha'_i\beta'_i, & i = j. \end{cases}$$

Lemma 2. φ is an automorphism of algebra A.

P r o o f. Having proved lemma 1, it suffices to show that $\varphi(x)\,\varphi(y) = \varphi(x\,y)$.

Let $\varphi(x)\,\varphi(y) = \sum_{i=1}^{44} \Delta_i e_i$, where $\Delta_i \in F$. Coefficients Δ_i are calculated according to symbols $[i', j']$ through formulae (3). We will prove that $\Delta_i = \delta_i'$ for every i :

1. $\Delta_1 = \Delta_2 = \Delta_3 = 0.$

2. $\Delta_4 = [3', 3'] = [3, 3] = \delta_4'$, $\Delta_5 = [2', 3'] = [1, 3] = \delta_7 = \delta_5'$,
$\Delta_6 = [1, 1] = \delta_9 = \delta_6'$, $\Delta_7 = [1', 3'] = [2, 3] = \delta_5 = \delta_7'$,
$\Delta_8 = [1', 2'] = [1, 2] = \delta_8 = \delta_8'$, $\Delta_9 = [1', 1'] = [2, 2] = \delta_6 = \delta_9'$.

3. $\Delta_{10} = -2\,[2', 4'] + [3', 5'] = -2\,[1, 4] + [3, 7] = \delta_{12} = \delta_{10}'$,
$\Delta_{11} = -\frac{1}{2}\,[2', 5'] + [3', 6'] = -\frac{1}{2}\,[1, 7] + [3, 9] = \delta_{16} = \delta_{11}'$,
$\Delta_{12} = -2\,[1', 4'] + [3', 7'] = -2\,[2, 4] + [3, 5] = \delta_{10} = \delta_{12}'$,
$\Delta_{13} = -[1', 5] + [2', 7'] = -[2, 7] + [1, 5] = -\delta_{13} = \delta_{13}'$,
$\Delta_{14} = -[1', 5'] + [3', 8'] = -[2, 7] + [3, 8] = \delta_{14} - \delta_{13} = \delta_{14}'$,
$\Delta_{15} = -2\,[1', 6'] + [2', 8'] = -2\,[2, 9] + [1, 8] = -2\,\delta_{17} = \delta_{15}'$,
$\Delta_{16} = -\frac{1}{2}\,[1', 7'] + [3', 9'] = -\frac{1}{2}\,[2, 5] + [3, 6] = \delta_{11} = \delta_{16}'$,
$\Delta_{17} = -\frac{1}{2}\,[1', 8'] + [2', 9'] = -\frac{1}{2}\,[2, 8] + [1, 6] = -\frac{1}{2}\,\delta_{15} = \delta_{17}'$.

From here and below we shall change for compact view symbols $[i', j']$ on $[i, j]$ according formulae (2). In doing so we shall use the following relations:
if $\alpha_i' = \alpha_k + \alpha_e$, $\alpha_j' = \alpha_m$ for distinguishing k, e, m, then

$$[i', j'] = (\alpha_k + \alpha_e)\,\beta_m + (\beta_k + \beta_e)\,\alpha_m = [k,m] + [e,m],$$
$$[i, j] = (\alpha_k + \alpha_e)\,(\beta_k + \beta_e) = [k,k] + [e,e] + [k,e];$$

if $\alpha_i' = \alpha_i + \alpha_j, \alpha_j' = \alpha_j$ for distinguishing i, j, then

$$[i', j'] = (\alpha_i + \alpha_j)\,\beta_j + (\beta_i + \beta_j)\,\alpha_j = [i, j] + 2\,[j, j].$$

4. $\Delta_{18} = -\frac{1}{2}\,[1, 12] + [3, 16] - 2\,[4, 9] + [7, 7] = \delta_{23} = \delta_{18}'$,
$\Delta_{19} = [2, 12] - [1, 10] - [3, 13] = -\delta_{19} = \delta_{19}'$,
$\Delta_{20} = -[1, 10] - [3, 13] = [3, 14] - 2\,[4, 8] + [5, 7] = \delta_{20} - \delta_{19} = \delta_{20}'$,
$\Delta_{21} = -2\,[2, 16] + [1, 13] - [1, 13] + [1, 14] - [7, 8] + 2\,[9, 5] = -2\delta_{24} = \delta_{21}'$,
$\Delta_{22} = [1, 13] - 2\,[3, 17] - [7, 8] + 2\,[9, 5] = -2\delta_{25} = \delta_{22}'$,
$\Delta_{23} = -\frac{1}{2}\,[2, 10] + [3, 11] - 2\,[4, 6] + [5, 5] = \delta_{18} = \delta_{23}'$,

$\Delta_{24} = \frac{1}{2}\,[2, 13] - \frac{1}{2}\,[2, 14] + [1, 11] - [6, 7] + \frac{1}{2}\,[5, 8] = -\frac{1}{2}\,\delta_{21} = \delta_{24}'$,
$\Delta_{25} = \frac{1}{2}\,[2, 13] - \frac{1}{2}\,[3, 15] - [7, 6] + \frac{1}{2}\,[5, 8] = -\frac{1}{2}\,\delta_{22} = \delta_{25}'$,
$\Delta_{26} = [2, 17] - \frac{1}{2}\,[1, 15] - 2\,[6, 9] + [8, 8] = \delta_{26} = \delta_{26}'$.

5. $\Delta_{27} = 2\,[2, 23] + [1, 19] - [1, 20] - 2\,[3, 24] - [7, 13] - 2\,[9, 10] + [8, 12] =$
$= -2\delta_{30} = \delta_{27}'$,
$\Delta_{28} = 2\,[2, 23] - [1, 19] + [1, 19] - [1, 20] - 2\,[3, 25] + 4\,[4, 17] - [7, 13] +$
$+ [7, 14] - 2\,[5, 16] = -2\delta_{31} = \delta_{28}'$,
$\Delta_{29} = 2\,[1, 24] - 2\,[1, 25] + 2[7, 17] - 2\,[9, 13] + 2\,[9, 14] - 2\,[8, 16] = -2\delta_{34} =$
$= \delta_{29}'$,
$\Delta_{30} = \frac{1}{2}\,[2, 19] - \frac{1}{2}\,[2, 19] + \frac{1}{2}\,[2, 20] - [1, 18] - \frac{1}{2}\,[3, 21] - \frac{1}{2}\,[5, 13] -$
$-\frac{1}{2}\,[8, 10] + [6, 12] = -\frac{1}{2}\,\delta_{27} = \delta_{30}'$,
$\Delta_{31} = -\frac{1}{2}\,[2, 19] + \frac{1}{2}\,[2, 20] - [1, 18] - \frac{1}{2}\,[3, 22] + [4, 15] + [7, 11] - \frac{1}{2}\,[5, 13] +$
$+ \frac{1}{2}\,[5, 13] - \frac{1}{2}\,[5, 14] = -\frac{1}{2}\delta_{28} = \delta_{30}'$,
$\Delta_{32} = [2, 24] - [2, 25] + \frac{1}{2}\,[1, 21] - \frac{1}{2}\,[1, 22] + \frac{1}{2}\,[7, 15] + 2\,[9, 11] + [5, 17] -$
$- \frac{1}{2}\,[8, 13] - 2\,[6, 16] = -\delta_{32} = \delta_{32}'$,
$\Delta_{33} = -[2, 25] + \frac{1}{2}\,[1, 21] + [3, 26] + \frac{1}{2}\,[7, 15] + 2\,[9, 11] - \frac{1}{2}\,[8, 13] +$
$+ \frac{1}{2}\,[8, 13] - \frac{1}{2}\,[8,14] = \delta_{33} - \delta_{32} = \delta_{33}'$,
$\Delta_{34} = \frac{1}{2}\,[2, 21] - \frac{1}{2}\,[2, 22] + \frac{1}{2}\,[5, 15] + [8, 11] - [6, 13] + [6, 13] - [6, 14] =$
$= -\frac{1}{2}\,\delta_{29} = \delta_{34}'$.

6. $\Delta_{35} = 2\,[1, 31] - 2\,[3, 34] - 2\,[7, 24] - 2\,[9, 19] + 2\,[9, 19] - 2\,[9, 20] + 2\,[8, 23] -$
$-2\,[12, 17] + 2\,[16, 13] = -2\delta_{39} = \delta_{35}'$,
$\Delta_{36} = [2, 31] + \frac{1}{2}\,[1, 28] - [3, 32] - \frac{1}{2}\,[7, 21] - 2\,[9, 18] - [5, 24] - \frac{1}{2}\,[8, 19] +$
$+ 2\,[6, 23] - \frac{1}{2}\,[12, 15] - [10, 17] + [13, 13] - 2\,[13, 13] + [13, 14] = -\delta_{36} = \delta_{36}'$,
$\Delta_{37} = [2, 30] - \frac{1}{2}\,[1, 27] + \frac{1}{2}\,[1, 28] - [3, 32] + [3, 33] - 2\,[4, 26] - \frac{1}{2}\,[7, 22] -$
$- [5,24] + [5, 25] - \frac{1}{2}\,[8, 19] + \frac{1}{2}\,[8, 19] - \frac{1}{2}\,[8, 20] + 2\,[6, 23] - 2\,[11, 16] -$
$- [10, 17] - 2\,[13, 13] + [13, 14] + [14, 14] + [13, 13] - [13, 14] = -\delta_{36} + \delta_{37} = \delta_{37}'$,

$\Delta_{38} = -[2, 34] + [1, 32] - [1, 32] + [1, 39] - [7, 26] - [9, 22] + [8, 25] + [16, 15] -$
$- [13, 17] + [13, 17] - [14, 17] = -\delta_{40} = \delta'_{38}$,

$\Delta_{39} = \frac{1}{2}[2, 28] - \frac{1}{2}[3, 29] - \frac{1}{2}[5, 21] - [8, 18] - [6, 19] + [6, 20] - \frac{1}{2}[10, 15] +$
$+ [13, 11] = -\frac{1}{2}\delta_{35} = \delta'_{39}$,

$\Delta_{40} = [2, 32] - [2, 33] - \frac{1}{2}[1, 29] + [5, 26] + \frac{1}{2}[8, 22] - 2[6, 25] + \frac{1}{2}[13, 15] -$
$- \frac{1}{2}[14, 15] + 2[17, 11] = -\delta_{38} = \delta'_{40}$.

7. $\Delta_{41} = -[2, 39] + [1, 36] - [1, 37] - [3, 40] - [7, 32] - [9, 27] + [9, 28] +$
$+ 2[5, 34] + [8, 30] - [8, 31] + [12, 26] + [16, 21] - [13, 24] + [13, 25] + [13, 24] -$
$- [14, 24] - [17, 19] + [17, 20] - [15, 23] = -\delta_{42} = \delta'_{41}$,

$\Delta_{42} = -[2, 36] + [2, 37] + [2, 36] - \frac{1}{2}[1, 35] - [3, 38] + [7, 29] - [5, 32] +$
$+ \frac{1}{2}[8, 27] - \frac{1}{2}[8, 28] - 2[6, 30] + 2[6, 31] - [10, 26] - \frac{1}{2}[13, 22] + \frac{1}{2}[13, 21] -$
$- \frac{1}{2}[14, 21] - 2[17, 18] + 2[11, 24] + \frac{1}{2}[15, 19] - \frac{1}{2}[15, 19] + \frac{1}{2}[15, 20] =$
$= -\delta_{41} = \delta'_{42}$,

$\Delta_{43} = -[2, 40] - [1, 38] + [9, 29] - [8, 32] + 2[6, 34] + [13, 26] + [17, 21] -$
$- [15, 24] = -\delta_{43} = \delta'_{43}$.

8. $\Delta_{44} = \frac{1}{2}[2, 42] + \frac{1}{2}[1, 41] - [3, 43] + \frac{1}{2}[7, 38] - \frac{1}{2}[9, 35] + \frac{1}{2}[5, 40] +$
$+ \frac{1}{2}[8, 36] - [6, 39] - \frac{1}{2}[16, 29] + \frac{1}{2}[13, 32] - \frac{1}{2}[13, 33] - \frac{1}{2}[13, 32] +$
$+ \frac{1}{2}[14, 32] - \frac{1}{2}[17, 27] - [11, 34] + \frac{1}{2}[15, 30] - \frac{1}{2}[19, 26] + \frac{1}{2}[24, 22] -$
$- \frac{1}{2}[25, 21] = -\delta_{44} = \delta'_{44}$.

This proves the lemma.

The purpose of this section of the article is to prove that $A \in N$. If we will choose the direct way of proof (i.e., prove $J(e_i, e_j, e_k) = 0$ for all i, j, k), it would require about $\bar{C}^3_{44} = C^3_{46} = 15180$ of elementary calculations. The following lemma shows that automorphism φ reduces a number of calculations.
Denote:

$k_i(x) = \alpha_i$ - i's coefficient for decomposition of x by basis,

$$Ann\, J(A) = \{\kappa \in A,\ J(x, A, A) = 0\}.$$

Lemma 3. Let B be a F-module is generated by $E \setminus \{e_i, e_j\}$, for some different i, j and

$$\text{(a)}\begin{cases}\varphi(e_i) = \alpha e_i,\\ \varphi(e_j) = \beta e_i + \gamma e_i,\end{cases} \quad or \quad \text{(b)}\begin{cases}\varphi(e_i) = \alpha e_j,\\ \varphi(e_j) = \beta e_i,\end{cases}$$

where $\alpha, \beta, \gamma \in F$ and α, β are not zero.
Then
1. $\forall\, u, v, w \quad k_i(J(u, v, w)) = 0 \Rightarrow k_j(J(u, v, w)) = 0$,
2. $e_j \in Ann\, J(A) \Rightarrow e_i \in Ann\, J(A)$.

P r o o f. 1,(*a*). We shall assume the contrary: there exists $u_0, v_0, w_0 \in A$, that

$$J(u_0, v_0, w_0) = \delta e_j + b, \text{ where } b \in B \text{ and } \delta \neq 0.$$

Then

$$\varphi(J(u_0, v_0, w_0)) = J(\varphi(u_0), \varphi(v_0), \varphi(w_0)) = \delta\,\varphi(e_j) + \varphi(b) = \delta\,\beta e_i + \delta\,\gamma\, e_j + \varphi(b).$$

in view of assumptions of the lemma we have $\delta\beta = 0$ and $\delta = 0$ or $\beta = 0$.
Contradiction has been obtained.
2,(*a*). Given $(e_j, A, A) = 0$ then $(\varphi(e_j), \varphi(A), \varphi(A)) = 0$, but
$\varphi(e_j) = \beta e_i + + \gamma e_j$, and $\varphi(A) = A$. Hence, $\beta(e_i, A, A) + \gamma(e_j, A, A) = 0$
and $(e_i, A, A) = 0$.
Case (*b*) is considered analogously. This proves the lemma.

Corollary.
1. Let x, y, z are general elements from A and $J = J(x, y, z)$, then

$$J = 0 \Leftrightarrow k_i(J) = 0,$$

for $i \in M_1 = \{10, 11, 14, 15, 18, 20, 21, 22, 26, 27, 28, 29, 33, 35, 36, 38, 41, 43, 44\}$.

2. If $e_i \in Ann\,(J)$, for $i \in M_2 = \{27, 28, 29, 32, 35, 36, 38\}$, then $e_i \in Ann\, J(A)$, for any $i \geq 27$.
The proof is a consequence of the lemma, definition of φ, and the fact that A is nilpotent-of-index 9.

Lemma 4. $e_i \in Ann\, J(A)$, for all $i \geq 27$.
P r o o f. In view of corollary of lemma 3 it suffices to prove lemma for $i \in M_2$.
Let us to enumerate all types of nonzero homogeneous elements of length 7 and 8 from A:
length 8, type: (3, 3, 2);
length 7, types: (2, 3, 2), (3, 2, 2), (3, 2, 1);
Let $u, v \in E$ and $J = J(e_i, u, v)$, $i \in M_2$. Comparing types $t(e_i)$ and $t(J)$, we will write down all pairs (u, v); when a proof that $J = 0$ is non-trivial :

1. e_{27} type (1, 2, 2): (a, a), (a, b), (a, ab), (b, a^2).
2. e_{28} type (1, 2, 2): the same as in case 1.
3. e_{29} type (1, 3, 1): (a, a), (a, c), $(a, a\,c)$.

4. e_{32} type (2, 2, 1): (a, b), (a, c), (b, c), $(a, b\ c)$, $(b, a\ c)$, $(c, a\ b)$.
5. e_{36} type (2, 2, 2): (a, b).
6. e_{38} type (2, 3, 1): (a, c).

Let us prove all these cases:

1. $e_{27}\, e_1\, e_1 + e_{27}e_1^{\,2} = -e_{42} + e_{42} = 0,$

$$e_{27}\, e_1\, e_2 + e_{27}\, e_2\, e_1 + e_{27}\,(e_1\, e_2) = \frac{1}{2}\, e_{41} - \frac{1}{2} e_{41} = 0,$$

$$e_{27}\, e_2\, e_8 + e_{27}\, e_9\, e_2 + e_{27}\,(e_1\, e_8) = \frac{1}{2}\, e_{44} - \frac{1}{2} e_{44} = 0,$$

$$e_{27}\, e_1\, e_9 + e_{27}\, e_9\, e_1 + e_{27}\,(e_2\, e_9) = -\frac{1}{2} e_{44} + \frac{1}{2}\, e_{44} = 0,$$

2. $2e_{28}\, e_1\, e_1 + e_{28}e_1^{\,2} = e_{42} - e_{42} = 0,$

$$e_{28}\, e_1\, e_2 + e_{28}\, e_2\, e_1 + e_{28}\,(e_1\, e_2) = -\frac{1}{2} e_{41} + \frac{1}{2}\, e_{41} = 0,$$

$$e_{28}\, e_1\, e_8 + e_{28}\, e_8\, e_1 + e_{28}\,(e_1\, e_8) = \frac{1}{4}\, e_{44} - \frac{1}{4}\, e_{44} = 0,$$

$$e_{28}\, e_2\, e_9 + e_{28}\, e_9\, e_2 + e_{28}\,(e_2\, e_9) = -\frac{1}{2} e_{44} + \frac{1}{2}\, e_{44} = 0,$$

$$2e_{29}\, e_1\, e_1 + e_{29}e_1^{\,2} = e_{43} - e_{43} = 0,$$

$$e_{29}\, e_1\, e_3 + e_{29}\, e_3\, e_1 + e_{29}\,(e_1\, e_3) = \frac{1}{2}\, e_{41} + \frac{1}{2}\, e_{41} - e_{41} = 0,$$

$$e_{29}\, e_1\, e_7 + e_{29}\, e_7\, e_1 + e_{29}\,(e_1\, e_7) = -\frac{1}{4}\, e_{44} + \frac{1}{2}\, e_{44} - \frac{1}{4}\, e_{44} = 0,$$

$$e_{32}\, e_1\, e_2 + e_{32}\, e_2\, e_1 + e_{32}\,(e_1\, e_2) = -e_{43} + e_{43} = 0,$$

$$e_{32}\, e_1\, e_3 + e_{32}\, e_3\, e_1 + e_{32}\,(e_1\, e_3) = -e_{42} + e_{42} = 0,$$

$$e_{32}\, e_2\, e_3 + e_{32}\, e_3\, e_2 + e_{32}\,(e_2\, e_3) = -e_{41} + e_{41} = 0,$$

$$e_{32}\, e_1\, e_5 + e_{32}\, e_5\, e_1 + e_{32}\,(e_1\, e_5) = -\frac{1}{2}\, e_{44} + \frac{1}{2}\, e_{44} = 0,$$

$$e_{32}e_2\, e_7 + e_{32}\, e_7\, e_2 + e_{32}\,(e_2\, e_7) = \frac{1}{2}\, e_{44} - \frac{1}{2}\, e_{44} = 0,$$

$$e_{32}\, e_3\, e_8 + e_{32}\, e_8\, e_3 + e_{32}\,(e_3\, e_8) = -\frac{1}{2}\, e_{44} + e_{44} - \frac{1}{2}\, e_{44} = 0,$$

$$e_{36}e_1\, e_2 + e_{36}\, e_2\, e_1 + e_{36}\,(e_1\, e_2) = \frac{1}{2}\, e_{44} - \frac{1}{2}\, e_{44} = 0,$$

$$e_{38}e_1\, e_3 + e_{38}\, e_3\, e_1 + e_{38}\,(e_1\, e_3) = e_{44} - \frac{1}{2}\, e_{44} - \frac{1}{2}\, e_{44} = 0,$$

This proves the lemma.
Let

$$z = \sum_{i=1}^{44} \gamma_i e_i, \qquad x\, y\, z = \sum_{i=1}^{44} d_i e_i,$$

where γ_i, $d_i \in \boldsymbol{F}$.

Let $\sigma = (\alpha\ \beta\ \gamma)$ is a cyclic permutation of symbols α, β, γ. Consider σ and $S = id + \sigma + \sigma^2$, as naturally defined homomorphisms on associative-commutative algebra from the set of free generators α_i, β_j, γ_k.

For instance:

$$\sigma\,(\alpha_1\, \beta_2\, \gamma_1 + \alpha_3\, \beta_1\, \gamma_2) = \gamma_1\, \alpha_2\, \beta_1 + \gamma_3\, \alpha_1\, \beta_2,$$

$$S\,(\alpha_i\, \beta_j\, \gamma_k) = \alpha_i\, \beta_j\, \gamma_k + \gamma_i\, \alpha_j\, \beta_k + \beta_i\, \gamma_j\, \alpha_k.$$

Note that

$$S \circ \sigma = S \circ \sigma^2 = S \tag{4}$$

Obviously,

$$z\, x\, y = \sum_{i=1}^{44} \sigma(d_i) e_i, \quad y\, z\, x = \sum_{i=1}^{44} \sigma^2(d_i) e_i \quad \text{and} \quad \boldsymbol{J}\,(x, y, z) = \sum_{i=1}^{44} S(d_i) e_i.$$

Let us find coefficient d_i. For this purpose we will define a symbol

$$\langle i, j\rangle = \begin{cases} \delta_i \gamma_j + \delta_j \gamma_i, & i \neq j, \\ \delta_i \gamma_i, & i = j. \end{cases}$$

Obviously that d_i identically coincides with δ_i, simply replacing brackets [,]
by brackets < , >.
For instance:

$$d_{10} = -2 <2, 4> + <3, 5> = -2(\gamma_2\, \delta_4 + \gamma_4\, \delta_2) + (\gamma_3\, \delta_5 + \gamma_5\, \delta_3) = -2[3, 3]\, \gamma_2 + [2, 3]\, \gamma_3,$$

since $\delta_2 = \delta_3 = 0$, $\delta_4 = [3, 3]$, $\delta_5 = [2, 3]$ from (2).

For compact notation equality $S\,(f) = S\,(g)$ will be written as: $f =_s g$.

Proposition 1. For different i, j, k:

$$[i, j]\, \gamma_k =_s [i, k]\, \gamma_j =_s [k, j]\, \gamma_i, \tag{5}$$

$$[i, j]\, \gamma_j =_s \frac{1}{2}\, [i, j]\, \gamma_i.$$

P r o o f. In view of (3) we have

$$S([i, j]\, \gamma_k) = S\,(\alpha_i\, \beta_j\, \gamma_k + \alpha_j\, \beta_i\, \gamma_k) = S\,(\alpha_j\, \beta_i\, \gamma_k) + S\,(\alpha_i\, \beta_j\, \gamma_k) =$$

$$= S \circ \sigma^2\,(\alpha_i\, \beta_j\, \gamma_k) + S \circ \sigma\,(\alpha_j\, \beta_i\, \gamma_k) = S\,(\beta_i\, \gamma_j\, \alpha_k) + S\,(\gamma_j\, \alpha_i\, \beta_k) = S\,([i, k]\, \gamma_j),$$

$$S\,([i, i]\, \gamma_j) = S\,(\alpha_i\, \beta_i\, \gamma_j) = \frac{1}{2}\,(S \circ \sigma + S \circ \sigma^2)\,(\alpha_i\, \beta_i\, \gamma_j) = \frac{1}{2}\, S\,(\gamma_i\, \alpha_i\, \beta_j + \beta_i\, \gamma_i\, \alpha_j) =$$

$$= \frac{1}{2}\, S\,([i, j]\, \gamma_i).$$

This proves the proposition.

Lemma 5. $A \in N$.

P r o o f. It suffices to prove that $S(d_i) = 0$ for all i. By reason of corollary of Lemma 3 it suffices to check it only for $i \in M_1$. In view of Lemma 4 we can assume that

$$\alpha_i = \beta_i = \gamma_i = 0 \quad \text{for} \quad i \geq 27.$$

From formulae (3) and (5) we obtain:

$d_{10} =_s -2\,[3,3]\,\gamma_2 + [2,3]\,\gamma_3 = 0$, $d_{11} =_s -\frac{1}{2}\,[2,3]\,\gamma_2 + [2,2]\,\gamma_3 = 0$,

$d_{14} =_s -[2,3]\,\gamma_1 + [1,3]\,\gamma_2 = 0$, $d_{15} =_s -2\,[2,2]\,\gamma_1 + [1,2]\,\gamma_2 = 0$,

$d_{18} =_s -\frac{1}{2}(-2\,[2,4] + [3,5])\,\gamma_2 + (-\frac{1}{2}[2,5] + [3,6])\,\gamma_3 - 2\,[3,3]\,\gamma_6 - 2\,[2,2]\,\gamma_4 +$

$+ [2,3]\,\gamma_5 = 0.$

$d_{20} =_s -(-2\,[1,4] + [3,7]\,\gamma_2 + (-[1,5] + [3,8]\,\gamma_3 - 2\,[3,3]\,\gamma_8 - 2\,[1,2]\,\gamma_4 +$

$+ [2,3]\,\gamma_7 + [1,3]\,\gamma_5 = 0,$

$d_{21} =_s -2\,(-\frac{1}{2}\,[2,5] + [3,6])\,\gamma_1 - (-[1,5] + [2,7])\,\gamma_2 + (-[1,5] + [3,8])\,\gamma_2 -$

$- [2,3]\,\gamma_8 - [1,2]\,\gamma_8 - [1,2]\,\gamma_5 + 2\,[2,2]\,\gamma_7 + 2\,[1,3]\,\gamma_6 = 0,$

$d_{22} =_s ([1,5] - [2,7]\,\gamma_2 + (-2\,[1,6] + [2,8])\,\gamma_3 - [2,3]\,\gamma_8 - [1,2]\,\gamma_5 + 2\,[2,2]\,\gamma_7 +$

$+ 2\,[1,3]\,\gamma_6 = 0,$

$d_{26} =_s -\frac{1}{2}(-2\,[1,6] + [2,8]\,\gamma_1 + (-\frac{1}{2}\,[1,8] + [2,9])\,\gamma_2 - 2\,[2,2]\,\gamma_9 - 2\,[1,1]\,\gamma_6 +$

$+ [1,2]\,\gamma_8 = 0,$

$d_{27} =_s 2\,(-\frac{1}{2}\,[2,10] + [3,11] - 2\,[4,6] + [5,5])\,\gamma_1 - (-[2,12] + [3,14] - 2\,[4,8] +$

$+ [5,7])\,\gamma_2 + (-2\,[1,11] - [2,13] + [2,14] - [5,8] + 2\,[6,7]\,\gamma_3 + [2.3]\,\gamma_{13} +$

$+(-[1,5] + [2,7])\,\gamma_5 - 2\,[2,2]\,\gamma_{12} - 2\,(-2\,[1,4] + [3,7])\,\gamma_6 + [1,2]\,\gamma_{10} +$

$+(-2\,[2,4] + [3,5])\,\gamma_8 = 0.$

$d_{28} =_s 2\,(-\frac{1}{2}\,[2,10] + [3,11] - 2\,[4,6] + [5,5]\,\gamma_1 - (-[2,12] + [3,14] - 2\,[4,8] +$

$+ [5,7]\,\gamma_2 + ([1,10] - [2,12] + [3,13]\,\gamma_2 + (-[2,13] + [3,15] - [5,8] + 2\,[6,7])\,\gamma_3 -$

$- 2\,[3,3]\,\gamma_{15} - 2\,(-2\,[1,6] + [2,8])\,\gamma_4 + [2,3]\,\gamma_{14} + (-[1,5] + [3,8])\,\gamma_5 - 2\,[1,3]\,\gamma_{11} - 2\,(-$

$\frac{1}{2}\,[2,5] + [3,6])\,\gamma_7 = 0,$

$d_{29} =_s (2\,[1,11] + [2,13] - [2,14] + [5,8] - 2\,[6,7] - [2,13] + [3,15] - [5,8] +$

$+ 2\,[6,7])\,\gamma_2 - [2,3]\,\gamma_{15} - (-2\,[1,6] + [2,8])\,\gamma_5 + 2\,[2,2]\,\gamma_{14} + 2\,(-[1,5] + [3,8])\,\gamma_6 - 2$

$[1,2]\,\gamma_{11} - 2\,(-\frac{1}{2}\,[2,5] + [3,6])\,\gamma_8 = 0,$

$d_{33} =_s \frac{1}{2}(-[2,13] + [3,15] - [5,8] + 2\,[6,7])\,\gamma_1 + (\frac{1}{2}\,[1,14] - [2,16] + [5,9] -$

$-\frac{1}{2}\,[7,8])\,\gamma_2 + (-\frac{1}{2}[1,15] + [2,17] - 2\,[6,9] + [8,8])\,\gamma_3 - [2,3]\,\gamma_{17} - (-\frac{1}{2}[1,8] +$

$+ [2,9]\,\gamma_5 + 2\,[2,2]\,\gamma_{16} + 2\,(-\frac{1}{2}[1,7] + [3,9])\,\gamma_6 + \frac{1}{2}[1,2]\,\gamma_{13} + \frac{1}{2}(-[1,5] +$

$+ [2,7])\,\gamma_8 - \frac{1}{2}\,[1,2]\,\gamma_{14} - \frac{1}{2}\,(-[1,5] + [3,8])\,\gamma_8 = 0,$

$d_{35} =_s (-2\,[1,18] - [2,19] + [2,20] - [3,29] + 2\,[4,15] - [5,14] + 2\,[7,11])\,\gamma_2 + (-$

$- [2,21] + [2,22] - [5,15] + 2\,[6,14] - 2\,[8,11]\,\gamma_3 + (-2\,[1,11] - [2,13] + [2,14] -$

$- [5,8] + 2\,[6,7]\,\gamma_5 + [2,3]\,\gamma_{21} + 2\,([1,10] - [2,12] + [3,13])\,\gamma_6 + 2\,[2,2]\,\gamma_{19} -$

$+2\,[2,2]\,\gamma_{20} - 2\,(-[2,12] + [3,14] - 2\,[4,8] + [5,7])\,\gamma_6 + 2\,[1,2]\,\gamma_{18} + 2\,(-\frac{1}{2}\,[2,10]$

$+ [3,11] - 2\,[4,6] + [5,5])\,\gamma_8 + (-2\,[2,4] + [3,5])\,\gamma_{15} + (-2\,[1,6] + [2,8])\,\gamma_{10} -$

$- 2\,(-\frac{1}{2}\,[2,5] + [3\ 63]\,\gamma_{13} - 2\,(-[1,5] + [2,7])\,\gamma_{11} = 0,$

$d_{36} =_s (-[1,18] - \frac{1}{2}\,[2,19] + \frac{1}{2}\,[2,20] - \frac{1}{2}\,[3,22] + [4,15] - \frac{1}{2}\,[5,14] -$

$- [7,11])\,\gamma_1 + (-\frac{1}{2}\,[1,20] + [2,23] - [3,25] + 2\,[4,17] - [5,16] - \frac{1}{2}\,[7,13] +$

$+ \frac{1}{2}\,[7,14])\,\gamma_2 + (-\frac{1}{2}\,[1,21] + \frac{1}{2}\,[1,22] - [2,24] + [2,25] - [5,17] + 2\,[6,16] -$

$-\frac{1}{2}\,[7,15] ++ \frac{1}{2}\,[8,13] - 2\,[9,11])\,\gamma_3 + [2,3]\,\gamma_{24} + (-\frac{1}{2}\,[1,14] + [2,16] - [5,9] +$

$+ \frac{1}{2}\,[7,8])\,\gamma_5 - 2\,[2,2]\,\gamma_{23} - 2\,(-\frac{1}{2}\,[1,12] + [3,16] - 2\,[4,9] + [7,7])\,\gamma_6 +$

$+ \frac{1}{2}\,[1,3]\,\gamma_{21} + \frac{1}{2}\,(-2\,[1,11] - [2,13] + [2,14] - [5,8] + 2\,[6,7])\,\gamma_7 +$

$+ \frac{1}{2}\,[1,2]\,\gamma_{19} + \frac{1}{2}\,([1,10] - [2,12] + [3,13])\,\gamma_8 + 2\,[1,1]\,\gamma_{18} - 2\,(-\frac{1}{2}\,[2,10] +$

$+ [3,11] - 2\,[4,6] + [5,5])\,\gamma_9 + (-2\,[2,4] + [3,5])\,\gamma_{17} + (-\frac{1}{2}\,[1,8] + [2,9])\,\gamma_{10} +$

$+ \frac{1}{2}\,(-2\,[1,4] + [3,7])\,\gamma_{15} + \frac{1}{2}\,(-2[1,6] + [2,8]\,\gamma_{12} + (-[1,5] + [2,7])\,\gamma_{13} -$

$- (-[1,5] + [2,7])\,\gamma_{14} - (-[1,5] + [3,8])\,\gamma_{13}\,]) = 0,$

$$d_{38} =_{s} \frac{1}{2} (- [2, 21] + [2, 22] - [5, 15] + 2 [6, 14] - 2 [8, 11]) \gamma_1 + (\frac{1}{2} [1, 21] -$$
$$- \frac{1}{2} [1, 22] + [2, 24] - [2, 25] + [5, 17] - 2 [6, 16] + \frac{1}{2} [7, 15] - \frac{1}{2} [8, 13] +$$
$$+ 2 [9, 11] + \frac{1}{2} [1, 22] - [2, 24] + [3, 26] - [5, 17] + 2 [6, 16] + \frac{1}{2} [8, 13] -$$
$$- \frac{1}{2} [8, 14]) \gamma_2 - [2, 3] \gamma_{26} - (- \frac{1}{2} [1, 15] + [2, 17] - 2 [6, 9] + [8, 8]) \gamma_5 +$$
$$+ 2 [2, 2] \gamma_{25} + 2 (- \frac{1}{2} [1, 13] + [3, 17] - [5, 9] + \frac{1}{2} [7, 8]) \gamma_6 - \frac{1}{2} [1, 2] \gamma_{22} -$$
$$- \frac{1}{2} (- [2, 13] + [3, 15] - [5, 8] + 2 [6, 7]) \gamma_8 - 2 (- \frac{1}{2} [2, 5] + 3, 6]) \gamma_{17} -$$
$$- 2 (- \frac{1}{2} [1, 8] + [2, 9]) \gamma_{11} - \frac{1}{2} (- [1, 5] + [2, 7]) \gamma_{15} - \frac{1}{2} (- 2 [1, 6] + [2, 8]) \gamma_{13} +$$
$$+ \frac{1}{2} (- [1, 5] + [3, 8]) \gamma_{15} + \frac{1}{2} (- 2 [1, 6] + [2, 8]) \gamma_{14} = 0,$$

$$d_{41} =_{s} (\frac{1}{2} [5, 21] + [6, 19] - [6, 20] + [8, 18] + \frac{1}{2} [10, 15] - [11, 13]) \gamma_1 + 2 [4, 26] -$$
$$- [5, 25] - \frac{1}{2} [7, 21] + \frac{1}{2} [7, 22] - \frac{1}{2} [8, 19] + \frac{1}{2} [8, 20] - 2 [9, 18] + 2 [11, 16] -$$
$$- \frac{1}{2} [12, 15] + [13, 14] - [14, 14]) \gamma_2 + (- [5, 26] + 2 [6, 25] - \frac{1}{2} [8, 22] - 2 [11, 17] -$$
$$- \frac{1}{2} [13, 15] + \frac{1}{2} [14, 15]) \gamma_3 + (- \frac{1}{2} [1, 21] + \frac{1}{2} [1, 22] - [2, 24] + [2, 25] - [5, 17] +$$
$$+ 2 [6, 16] - \frac{1}{2} [7, 15] + \frac{1}{2} [8, 13] - 2 [9, 11]) \gamma_5 + (- [1, 19] + [1, 20] - 2 [2, 23] +$$
$$+ 2 [3, 24] + [7, 13] - [8, 12] + 2 [9, 10] - [1, 20] + 2 [2, 23] - 2 [3, 25] + 4 [4, 17] -$$
$$- 2 [5, 16] - [7, 13] + [7, 14]) \gamma_6 + ([2, 21] - [2, 22] + [5, 15] - 2 [6, 14] + 2 [8, 11]) \gamma_7$$
$$+ ([1, 18] + \frac{1}{2} [2, 19] - \frac{1}{2} [2, 20] + \frac{1}{2} [3, 22] - [4, 15] + \frac{1}{2} [5, 14] - [7, 11] - [1, 18] +$$
$$+ \frac{1}{2} [2, 20] - \frac{1}{2} [3, 21] - \frac{1}{2} [5, 13] + [6, 12] - \frac{1}{2} [8, 10]) \gamma_8 + (- 2 [2, 4] + [3, 5]) \gamma_{24} + (- \frac{1}{2}$$
$$[1, 15] + [2, 17] - 2 [6, 9] + [8, 8]) \gamma_{10} + ([2, 5] - 2 [3, 6]) \gamma_{24} + ([1, 14] -$$
$$- 2 [2, 16] + 2 [5, 9] - [7, 8]) \gamma_{11} + (- \frac{1}{2} [2, 13] + \frac{1}{2} [3, 15] - \frac{1}{2} [5, 8] + [6, 7] +$$
$$+ [1, 11] + \frac{1}{2} [2, 13] - \frac{1}{2} [2, 14] + \frac{1}{2} [5, 8] - [6, 7]) \gamma_{13} + (\frac{1}{2} [1, 5] - \frac{1}{2} [2, 7]) \gamma_{21} +$$
$$+ (- \frac{1}{2} [1, 5] + \frac{1}{2} [2, 7]) \gamma_{22} + (- \frac{1}{2} [1, 5] + \frac{1}{2} [3, 8]) \gamma_{21} + (- [1, 11] - \frac{1}{2} [2, 13] +$$
$$+ \frac{1}{2} [2, 14] - \frac{1}{2} [5, 8] + [6, 7]) \gamma_{14} + ([1, 6] - \frac{1}{2} [2, 8]) \gamma_{20} + (\frac{1}{2} [2, 12] - \frac{1}{2} [3, 14] +$$
$$+ [4, 8] - \frac{1}{2} [5, 7]) \gamma_{15} + (- [1, 8] + 2 [2, 9]) \gamma_{18} + (- [2, 10] + 2 [3\ 11] - 4 [4\ 6] +$$
$$+ 2 [5, 5]) \gamma_{17} = 0,$$

$$d_{43} =_{s} (- [5, 26] + 2 [6, 25] - \frac{1}{2} [8, 22] - 2 [11, 17] - \frac{1}{2} [13, 15] + \frac{1}{2} [14, 15]) \gamma_1 +$$
$$+ ([7, 26] - [8, 25] + [9, 22] + [14, 17] - [15, 16]) \gamma_2 + (- \frac{1}{2} [1, 21] + \frac{1}{2} [1, 22] -$$
$$-[2, 24] + [2, 25] - [5, 17] + 2 [6, 16] - \frac{1}{2} [7, 15] + \frac{1}{2} [8, 13] - 2 [9, 11]) \gamma_8 +$$
$$+ (2 [1, 24] - 2 [1\ 25] + 2 [7, 17] - 2 [8, 16] - 2 [9, 13] + 2 [9, 14]) \gamma_6 + ([2, 21] -$$
$$- [2, 22] + [5, 15] - 2 [6, 14] + 2[8, 11]) \gamma_9 + ([1, 5] - [2, 7]) \gamma_{26} + (\frac{1}{2}[1, 15] - [2, 17] + + 2$$
$$[6, 9] - [8, 8]) \gamma_{13} + (- 2 [1, 6] + [2, 8]) \gamma_{24} + (- \frac{1}{2} [1, 14] + [2, 16] - [5, 9] +$$
$$+ \frac{1}{2} [7, 8]) \gamma_{15} + \frac{1}{2} [1, 8] - [2, 9]) \gamma_{21} + (2 [1, 11] + [2, 13] - [2, 14] + [5, 8] -$$
$$- 2 [6, 7]) \gamma_{17} = 0,$$

$$2d_{44} =_{s} - ([10, 26] - 2 [11, 24] - \frac{1}{2} [13, 22] + \frac{1}{2} [13, 21] + \frac{1}{2} [14, 21] - \frac{1}{2} [15, 20] +$$
$$+ 2 [17, 18]) \gamma_1 - (- [12, 26] - [13, 25] + [14, 24] + 15, 23] - [16, 21] + [17, 19] -$$
$$- [17, 20]) \gamma_2 + 2 (- [13, 26] + [15, 24] - [17, 21]) \gamma_3 - ([7, 26] - [8, 25] + [9, 22] +$$
$$+ [14, 17] - [15, 16]) \gamma_5 + 2([7, 24] - [8, 23] + [9, 20] + [12, 17] - [13, 16]) \gamma_6 -$$
$$- (- [5, 26] + 2 [6, 25] - \frac{1}{2} [8, 22] - 2 [11, 17] - \frac{1}{2} [13, 15] + \frac{1}{2} [14, 15]) \gamma_7 - [5, 24] -$$
$$- 2 [6, 23] + + \frac{1}{2} [7, 21] + \frac{1}{2}[8, 19] + 2 [9, 18] + [10, 17] + \frac{1}{2} [12, 15] + [13, 13] -$$
$$- [13, 14]) \gamma_8 + ([5, 21] + 2 [6, 19] - 2 [6, 20] + 2 [8, 18] + [10, 15] - 2 [11, 13]) \gamma_9 +$$
$$+ 2 (- [1, 24] + [1, 25] - [7, 17] + [8, 16] + [9, 13] - [9, 14]) \gamma_{11} + (\frac{1}{2} [1, 22] -$$
$$- [2, 24] + [3, 26] - [5, 17] + 2 [6, 16] + \frac{1}{2} [8, 13] - \frac{1}{2} [8, 14]) \gamma_{13} - (- \frac{1}{2} [1, 21] +$$
$$+ \frac{1}{2} [1, 22] - [2, 24] + [2, 25] - [5,17] + 2 [6, 6] - \frac{1}{2} [7, 15] + \frac{1}{2} [8, 13] -$$
$$- 2 [9, 11]) \gamma_{14} - (- \frac{1}{2} [1, 19] + \frac{1}{2}[1,20] - [2, 23] + [3, 24] ++ \frac{1}{2} [7, 13] - \frac{1}{2} [8, 12] +$$
$$+ [9, 10]) \gamma_{15} + (- [2, 21] + [2, 22] - [5, 15] + 2 [6, 14] - 2 [8, 11]) \gamma_{16} + (2 [1,18] -$$
$$- [2, 20] + [3, 21] + [5, 13] - 2 [6, 12] + [8, 10]) \gamma_{17} + ([1, 10] - [2, 12] + [3, 13]) \gamma_{26} +$$
$$+ (- \frac{1}{2} [1, 15] + [2, 17] - 2 [6, 9] + [8, 8]) \gamma_{19} + (- 2[1, 11] - [2, 13] + [2, 14] - [5, 8] +$$
$$+ 2[6, 7]) \gamma_{25} + (- \frac{1}{2} [1, 13] + [3, 17] -[5, 9] + \frac{1}{2} [7, 8]) \gamma_{21} - (- [2, 13] + [3, 15] -$$
$$- [5, 8] + 2 [6, 7]) \gamma_{24} - (- \frac{1}{2} [1, 14]+ [2, 16] - [5, 9] + \frac{1}{2} [7, 8]) \gamma_{22} = 0.$$

This proves the lemma.

3. **Main results.** To prove algebra A is exceptional, it suffices to check that Glennie s-identity [1]:

$$G_8(x, y, z) = 2z\, U_{x\cdot y},\, z\, U_x U_y - 2\, z\, U_{x\cdot y},\, zU_y U_x + (x\cdot y)\, U_z U_y U_x - (x\cdot y)\, U_z U_x U_y,$$

is not identity in A. Note that for variety N we can write:

$$uU_{v,w} = u\cdot v\cdot w + u\cdot w\cdot v - u\cdot(w\cdot v) = -2\, u\cdot(w\cdot v).$$

Hence, in the algebra A:

$$zU_xU_y = 4\, z\cdot x^2\cdot y^2,$$

$$zU_{x\cdot y, 4z\cdot x^2\cdot y^2} = -8\, z\cdot x^2\cdot y^2\cdot(x\cdot y)\cdot z,$$

$$(x\cdot y)\, U_zU_xU_y = -8\,(x\cdot y)\cdot z^2\cdot x^2\cdot y^2.$$

Consequently,

$$-\frac{1}{8}G_8(x, y, z) = 2\, z\cdot x^2\cdot y^2\cdot(x\cdot y)\cdot z - 2\, z\cdot y^2\cdot x^2\cdot(x\cdot y)\cdot z + (x\cdot y)\cdot z^2\cdot y^2\cdot x^2 -$$
$$-(x\cdot y)\cdot z^2\cdot x^2\cdot y^2.$$

Let $x = a,\ y = b,\ z = c$, then

$$-\frac{1}{8}\, G_8(a, b, c) = 2e_3\cdot e_9\cdot e_6\cdot e_8\cdot e_3 - 2e_3\cdot e_6\cdot e_9\cdot e_8\cdot e_3 + e_8\cdot e_4\cdot e_6\cdot e_9 - e_8\cdot e_4\cdot e_9\cdot e_6 =$$
$$= 2e_{16}\cdot e_6\cdot e_8\cdot e_3 - 2e_{11}\cdot e_9\cdot e_8\cdot e_3 - 2e_{20}\cdot e_6\cdot e_9 + 2e_{20}\cdot e_9\cdot e_6 = (4e_{32} + 4e_{33})\cdot e_8\cdot e_3 +$$
$$+ 4e_{32}\cdot e_8\cdot e_3 + 4e_{35}\cdot e_9 + 2e_{39}\cdot e_6 = 8e_{43}\cdot e_3 + 2e_{44} + 2e_{44} = 8e_{44} + 4e_{44} = 12e_{44} \neq 0.$$

Consequently, algebra A is exceptional and the following Theorem has been proved.

Theorem 1. Variety N is not special and $SN \underset{\neq}{\subset} N$.

For the construction of the nonspecial nill of index 3 Jordan algebra, which is a homomorphic image of the special one, we need to remind some definitions and results from [4].

Let I be an ideal of a special Jordan algebra J with associative enveloping algebra Ass. We shall denote by

$$\bar{I} = \hat{I} \cap J$$

the intersection J and the ideal

$$\hat{I} = (I)_{Ass}$$

of algebra Ass generated by the set I. We shall call $\bar{I}$ the Ass-closure of I in J, and say I is Ass-closed if $\bar{I} = I$. We note that for all ideals $I \subseteq \bar{I}$, so I is closed if and only if

$$\bar{I} \subseteq I.$$

Cohn's criterion [4]. J/I is special $\Leftrightarrow$ I is Ass - closed, for Ass the special universal associative enveloping algebra Ass.

Let $J[X]$, $SJ[X]$, $Ass[X]$ be free Jordan, free special Jordan, free associative algebras.

A set

$$S = (f, k_1, k_2, \ldots, k_n)$$

of elements of the free special Jordan algebra $SJ[X]$ is called a Cohn collection if f belongs to the $Ass[X]$- closure of the ideal K in $SJ[X]$ generated by the elements $k_1, k_2, \ldots, k_n$ but does not belongs to K, i.e.

$$f \in \bar{K} \text{ but } f \notin K = (k_1, k_2, \ldots, k_n)_{SJ[X]}.$$

The Cohn quasi-identity of S is

$$k_1 = 0, k_2 = 0, \ldots, k_n = 0 \Rightarrow f = 0,$$

a Cohn identity for S is an identity of the form

$$f - \sum_{i=1}^{n} M_i(k_i) = 0,$$

for $M_1, \ldots, M_n$ multiplication operators.

In view of the Theorem 1.1. [4], a special variety of Jordan algebras must have a Cohn identity for S as one of its defining identities for each Cohn collection S.

Now we are going to prove that SN is not a special variety of Jordan algebras.

Let K be the ideal of $SJ[X]$ which is generated by k^2, where $k = [x_1, x_2]$. From proof of the Theorem 2.3. [7] it follows that

$$f = \{k^2 x_1 x_3 x_4\} \in SJ[X] \quad\text{and}\quad f \notin K.$$

Hence, (f, k^2) is a Cohn collection. We shall first find the Jordan expression for f.
We shall write $u \equiv v$ if $u - v \in SJ[X]$ and $u \equiv_K v$ if $u - v \in K$.
The following identities are fulfilled in each associative algebra:

$$[a,b]\cdot c + [a,c]\cdot b = [a, b\cdot c],$$

$$[a\cdot b, c] = [a, b\cdot c] + [b, a\cdot c], \tag{6}$$

$$4aDb,c = [a,[b,c]],$$

$$ab = a\circ b + \frac{1}{2}[a,b].$$

Moreover, $\{x_1x_2x_3x_4\}$ is a skew-symmetric function modulo $SJ[X]$; in addition,

$$\{x_1x_2x_3x_4\} \equiv \frac{1}{2}[x_1, x_2]\cdot[x_3, x_4] \equiv \frac{1}{2}[[x_1, x_2]\cdot x_3, x_4].$$

We denote by $x = x_1, y = x_2, z = x_3, t = x_4$. Using these properties and identities (6), we have

$$k^2 = [x \cdot [x, y], y] - [[x, y], y] \cdot x = \frac{1}{2}[[x^2, y], y] + 4yDx, y \cdot x =$$
$$= -2yDx^2, y + 4yDx, y \cdot x = -2x^2 \cdot y \cdot y + 2x^2 \cdot y^2 + 4x \cdot y \cdot y \cdot x - \tag{7}$$
$$-4y^2 \cdot x \cdot x \equiv_I -6x^2 \cdot y \cdot y - 6y^2 \cdot x \cdot x \equiv_I -12x^2 \cdot y \cdot y.$$

Then

$$f \equiv_K -\frac{1}{2}[k^2, z] \cdot [x, t] = -[k, z] \cdot k \cdot [x, t] = 4zDx, y \cdot k \cdot [x, t] =$$
$$= 4kD(zDx, y), [x, t] + 4k \cdot [x, t] \cdot zDx, y.$$

Now

$$4kD(zDx, y), [x, t] = [k, [(zDx, y), [x, t]]] = -16zDx, yDx, tDx, y,$$

and by (6)

$$4k \cdot [x, t] \equiv_I -24x^2 \cdot y \cdot t - 24x^2 \cdot t \cdot y.$$

Finally,

$$f \equiv_I -16zDx, yDx, tDx, y - 24(x^2 \cdot y \cdot t + x^2 \cdot t \cdot y) \cdot (zDx, y) \equiv$$
$$\equiv_I -16zDx, yDx, tDx, y + 24t \cdot y \cdot x^2 \cdot (zDx, y).$$

Hence, a Cohn identity for (f, k^2) in the variety *SN* has the form

$$2zDx, yDx, tDx, y - 3t \cdot y \cdot x^2 \cdot (zDx, y) \equiv_K 0, \tag{8}$$

Let **A** be an arbitrary algebra and **I** is an ideal in **A.** We shall denote an identity of the form

$$f - \sum_{i=1}^{n} M_i(k_i) = 0,$$

for $M_1, \dots, M_n$ multiplication operators and $k_i \in I$ by $f \equiv_I 0$.

Lemma 7. If a Cohn identity for (f, k^2) is valid in the variety *SN* then

$$[x, y] \circ ((zDx, y) \circ x) \equiv_{\overline{K}} 0, \tag{9}$$

is an identity in $USJ_3[x, y, z]$. Where $\overline{K}$ is the ideal of $USJ_3[x, y, z]$ which is generated by Rk^2, where $k = [x, y]$.

Proof. By (7) we have in the algebra $USJ_3[x, y, z]$

$$-2R((zDx, y) \cdot x)[Rx, Ry] + 2R(zDx, y)Rx[Rx, Ry] + 3RyRx^2R(zDx, y) \equiv_{\overline{K}} 0.$$

We set $a = Rx, b = Ry, c = Rz$ in (8). Then, in view of identity

$$R(x \cdot y) = -2Rx \circ Ry \tag{10}$$

in the algebra $USJ_3[X]$, we have $Rk^2 = -8[a, b]^2$ and

$$2((cDa, b) \circ a)[a, b] + (cDa, b)a[a, b] - 3ba^2(cDa, b) \equiv_{\overline{K}} 0.$$

It is easy to see, that any Jordan polynomial in $USJ_3[x, y, z]$ from a, b, c of the type [3,2,1] belongs to $\overline{K}$. By (6) we have:

$$((cDa, b) \circ a)[a, b] \equiv_{\overline{K}} [a, b] \circ ((cDa, b) \circ a).$$

$$(cDa, b)a[a, b] \equiv_{\overline{K}} ((cDa, b) \circ a)[a, b] + \frac{1}{2}[(cDa, b), a][a, b] \equiv_{\overline{K}}$$
$$\equiv_{\overline{K}} [a, b] \circ ((cDa, b) \circ a) + \frac{1}{4}[[(cDa, b), a], [a, b]].$$

$$[a, b] \circ ((cDa, b) \circ a) = \frac{1}{4}[c, [a, b]] \circ a \circ [a, b] \equiv_{\overline{K}} \frac{1}{4}[c, [a, b]] \circ a \circ [a, b] -$$
$$-\frac{1}{4}[c, [a, b]] \circ [a, b] \circ a =$$
$$= \frac{1}{16}[[c, [a, b]], [a, [a, b]]] \equiv_{\overline{K}} \frac{1}{16}[[[c, [a, b]], a], [a, b]] = \frac{1}{4}[[(cDa, b), a], [a, b]].$$

Hence,

$$(cDa, b)a[a, b] \equiv_{\overline{K}} 2[a, b] \circ ((cDa, b) \circ a).$$

Furthermore

$$ba^2(cDa, b) = (b \circ a^2)(cDa, b) + \frac{1}{2}[b, a^2](cDa, b) \equiv_{\overline{K}}$$
$$\equiv_{\overline{K}} \frac{1}{2}[(b \circ a^2), (cDa, b)] + \frac{1}{2}[b, a^2] \circ (cDa, b).$$

We note that

$$aDa,b = a^2 \circ b - a \circ b \circ a = \frac{3}{2} a^2 \circ b,$$

$$cDa,bDa,b = c \circ a \circ b \circ a \circ b + c \circ b \circ a \circ b \circ a -$$

$$-c \circ a \circ b \circ b \circ a - c \circ b \circ a \circ a \circ b \equiv_{\bar{K}} \frac{1}{2} c \circ a^2 \circ b \circ b + \frac{1}{2} c \circ b^2 \circ a \circ a \equiv_{\bar{K}} 0,$$

then,

$$\frac{1}{2}\left[(a^2 \circ b),(cDa,b)\right] = \frac{1}{3}\left[(aDa,b),(cDa,b)\right] = -\frac{1}{3}\left[a,b\right] \circ ((cDa,b) \circ a),$$

$$\frac{1}{2}\left[b,a^2\right] \circ (cDa,b) = \left[b,a\right] \circ a \circ (cDa,b) = \left[b,a\right] \circ (a \circ (cDa,b)) +$$

$$+aD\left[b,a\right],(cDa,b) \equiv_{\bar{K}} -\left[a,b\right] \circ ((cDa,b) \circ a).$$

Finally, we obtain

$$8\left[a,b\right] \circ ((cDa,b) \circ a) \equiv_{\bar{K}} 0.$$

This proves the lemma.

Let $f(x,y,z) = [x,y] \circ A + [B,x] + [A,y]$ be a homogeneous polynomial of the type [3,2,1], where $A,B,C \in SJ[x,y,z]$.

Proposition 2. If $f(x,y,z)$ is an identity in the algebra $Ass[x,y,z]$, then

$$A = \alpha[z \circ x,[x,y]], \qquad B = -\alpha[z \circ x \circ [x,y], y], \qquad C = \alpha[z \circ x \circ [x,y], x],$$

where $\alpha \in F$.

Proof. By the Cohn's Theorem [1] $H[x,y,z] = SJ[x,y,z]$. Hence,

$$A = \alpha_1\{xyzx\} + \alpha_2\{x^2zy\} + \alpha_3\{x^2yz\} + \alpha_4\{xzxy\} + \alpha_5\{xyxz\} + \alpha_6\{yx^2z\}, \qquad \alpha_i \in F,$$

$$B = \beta_1\{xy^2zx\} + \beta_2\{xyzyx\} + \beta_3\{x^2zy^2\} + \beta_4\{x^2yzy\} + \beta_5\{xyxzy\} +$$

$$+\beta_6\{xzxy^2\} + \beta_7\{xzyxy\} + \beta_8\{xyzxy\} + \beta_9\{xy^2xz\} + \beta_{10}\{x^2y^2z\} + \beta_{11}\{xyxyz\} +$$

$$+\beta_{12}\{yx^2zy\} + \beta_{13}\{yxzxy\} + \beta_{14}\{yx^2yz\} + \beta_{15}\{y^2x^2z\} + \beta_{16}\{yxyxz\}, \qquad \beta_i \in F,$$

$$C = \gamma_1\{x^2yzx\} + \gamma_2\{x^2zyx\} + \gamma_3\{xyxzx\} + \gamma_4\{x^2zxy\} + \gamma_5\{x^3zy\} +$$

$$+\gamma_6\{xzx^2y\} + \gamma_7\{x^3yz\} + \gamma_8\{x^2yxz\} + \gamma_9\{xyx^2z\} + \gamma_{10}\{yx^3z\}, \qquad \gamma_i \in F.$$

Substituting the expression of A,B,C in $f(x,y,z) = 0$ and equating the coefficients of the linearly independent monomials in $Ass[x,y,z]$, we obtain simple system of equations from which follow the equalities :

$$\alpha_1 = \alpha_5 = -\alpha_4 = -\alpha_6, \qquad \alpha_2 = \alpha_3 = 0,$$

and

$$A = \alpha\{xyzx\} - \alpha\{xzxy\} + \alpha\{xyxz\} - \alpha\{yx^2z\} =$$

$$= 2\alpha\{xy(z \circ x)\} - 2\alpha\{yx(z \circ x)\} = 2\alpha[[x,y], x \circ z],$$

$$where \quad \alpha = \alpha_1.$$

Analogously we obtain the expression for B,C. The proposition is proved.

Let us consider the algebra $SJ_3[x,y,z]$. By Theorem 2 [5], it is a special Jordan algebra. By Cohn's criterion, the universal associative envelope $A_3 = Ass_3[x,y,z]$ is isomorphic to the quotient algebra $Ass[x,y,z] / \hat{J}$, where $\hat{J}$ is an ideal of $Ass[x,y,z]$ generated by the set $J = J(SJ[x,y,z])$.

Lemma 8. The identity (8) is not valid in A_3.
Proof. . Let I be the ideal of algebra $Ass[x,y,z]$ generated by the set J and $k^2 = [x,y]^2 = 6x^2 \circ y^2$. We shall assume the contrary :

$$w = [x,y] \circ ((zDx,y) \circ x) \in \hat{I}.$$

By the lemma 4 [6],

$$w = u_0 + [u_1,x] + [u_2,y] + [u_3,z] + [u_4,x] \circ y + [u_5,x] \circ z + +[u_6,y] \circ z + [[u_7,x] \circ y, z],$$

where $u_i \in J$ or $u_i \in K$ and $i = 0,1,\dots,7$. In view of the fact that w is a skew-symmetric element with respect to the standard involution $*$ of the algebra $Ass[X]$, we can assume that $u_0 = u_7 = 0$.
We shall denote by U the F- module generated by the elements $[u_1,x]$, $[u_1,x]$, $[x,y] \circ u$, for $u \in J$.
Now we consider $[u_3,z]$. The element u_3 has the type [3,2,0], hence $u_3 \in J$. But, for any a from $SJ[X]$

$$
\begin{aligned}
&[a^3, z] = [a, z \circ a^2] + [a^2, z \circ a] = [a, z \circ a^2] + 2[a, z \circ a \circ a] = \\
&= [a, (2z \circ a \circ a + z \circ a^2)]
\end{aligned} \tag{11}
$$

Hence, $[u_3, z] \equiv_U 0$.
Furthermore, by (6), $[u_4, x] \circ y = [u_4 \circ y, x] + [x, y] \circ u_4 \equiv_U [x, y] \circ u_4$.
Since the type of the element u_4 is [2,1,1] then $u_4 \in J$ and $[u_4, x] \circ y \equiv_U 0$.
Now in the case $[u_5, x] \circ z$, we have u_5 that has type [2,2,0], hence
$u_5 = \alpha x^2 \circ y^2 + a, a \in J$, $\alpha \in F$. By (6), (11) we have

$$
\begin{aligned}
&[u_5, x] \circ z = [u_5, x \circ z] - [u_5, z] \circ x \equiv_U \alpha[x^2 \circ y^2, x \circ z] - \\
&-\alpha[x^2 \circ y^2, z] \circ x \equiv_U 2\alpha[x, z \circ x \circ y^2 \circ x] - 2\alpha[x, z \circ y^2 \circ x] \circ x - \\
&-2\alpha[y, z \circ x^2 \circ y] \circ x \equiv_U 2\alpha[x, (z \circ x \circ y^2 \circ x - z \circ y^2 \circ x \circ x)] - \\
&-2\alpha[y, z \circ x^2 \circ y \circ x] + 2\alpha[y, x] \circ (z \circ x^2 \circ y) \equiv_U \\
&\equiv_U \alpha[x, z \circ y^2 \circ x^2] - 2\alpha[y, z \circ x^2 \circ y \circ x] - 2\alpha[x, y] \circ (z \circ x^2 \circ y).
\end{aligned}
$$

Furthermore, the element u_6 has type [3,1,0], hence $u_6 \in J$. And by (6), (11) we have

$$[u_6, y] \circ z = [u_6, y \circ z] - [u_6, z] \circ y \equiv_U 0.$$

Finally, the element u_2 has type [2,2,1], hence $u_2 \equiv_J \beta x^2 \circ y^2 \circ z$, where $\beta \in F$ ..
Analogously, u_3 has type [3,1,1] and, hence $u_3 \in J$.
Consequently, in the algebra $Ass[x, y, z]$, we have an identity

$$
\begin{aligned}
&[x, y] \circ ((zDx, y) \circ x - 2\alpha(z \circ x^2 \circ y) + v_1) + +[(\alpha z \circ y^2 \circ x^2 - \beta x^2 \circ y^2 \circ z + v_2), x] + \\
&+[(-2\alpha z \circ x^2 \circ y \circ x + v_3), y] = 0.
\end{aligned}
$$

From Proposition 2 we have the identities in the algebra $SJ[x, y, z]$:

$$
\begin{aligned}
&(zDx, y) \circ x - 2\alpha(z \circ x^2 \circ y) \equiv_J \gamma[z \circ x, [x, y]], \\
&\alpha(z \circ y^2 \circ x^2) - \beta(x^2 \circ y^2 \circ z) \equiv_J -\gamma[z \circ x \circ [x, y], y], \\
&-2\alpha(z \circ x^2 \circ y \circ x) \equiv_J \gamma[z \circ x \circ [x, y], x],
\end{aligned}
$$

where $\gamma \in F$.

By (6), (7) we have :

$$
\begin{aligned}
&[z \circ x, [x, y]] = 4(z \circ x)Dx, y, \\
&[z \circ x \circ [x, y], y] = [z \circ x, y] \circ [x, y] + [[x, y], y] \circ (z \circ x) \equiv_J \\
&\equiv_J 6(z \circ x \circ x) \circ y^2 + 6y^2 \circ x \circ (z \circ x) \equiv_J 3x^2 \circ y^2 \circ z, \\
&[z \circ x \circ [x, y], x] = [z \circ x, x] \circ [x, y] + [[x, y], x] \circ (z \circ x) \equiv_J \\
&\equiv_J -6(z \circ x \circ y) \circ x^2 - 6x^2 \circ y \circ (z \circ x) \equiv_J 0.
\end{aligned}
$$

In view of speciality of the algebra $SJ_3[x, y, z]$ we have the identities in the algebra $SJ_3[x, y, z]$:

$$
\begin{aligned}
&(zDx, y) \circ x - 2\alpha(z \circ x^2 \circ y) = 4\gamma(z \circ x)Dx, y, \\
&\alpha(z \circ y^2 \circ x^2) - \beta(x^2 \circ y^2 \circ z) = -3\gamma x^2 \circ y^2 \circ z, \\
&-2\alpha(z \circ x^2 \circ y \circ x) = 0,
\end{aligned} \tag{12}
$$

From the table of multiplication of the algebra **A**, we have $Ann(A) = Fe_{44}$.
Let

$$B \cong A \,/\, Ann(A),$$

then the algebra **B** has the basis $e_1, e_2, \ldots, e_{43}$ with the same table of multiplication with replacing e_{44} by 0. It is clear that **B** is a nilpotent of index 8 Jordan algebra hence, by Glennie Theorem, all s-identity are valid in **B.** Hence **B** is a homomorphic image of $SJ_3[X]$ and (12) are the identities in **B.**
Setting $x = e_1, y = e_2, z = e_3$ in (12), we obtain :
$\alpha e_3 \cdot e_9 \cdot e_2 \cdot e_1 = -\alpha e_{34} = 0$ and $\alpha = 0$,
$(e_3De_1, e_2) \cdot e_1 = 4\gamma(e_3 \cdot e_1)De_1, e_2$ or $-\frac{1}{2}e_{24} - e_{25} = -2\gamma e_{24} + 2\gamma e_{25}$.
From this follows the contradiction : $\gamma = -\frac{1}{2}$ and $\gamma = \frac{1}{4}$. This proves the lemma.

Let us remained that a linear mapping ρ of $J \in N$ in an associative algebra A is called a multiplication specialisation if and only if

$$(a \cdot b)^\rho = -2a^\rho \circ b^\rho .$$

Let J be a special algebra from N, and $(U(J), \rho)$, $(A(J), \sigma)$ be the universal multiplication and associative envelope algebras.

Proposition 3. The algebra $A(J)$ is a homomorphic image of $U(J)$.

Proof. Let us prove that $\pi = -\frac{1}{2}\sigma$ is a multiplication specialisation.

We have

$$(a \cdot b)^{\pi} = -\frac{1}{2}(a \cdot b)^{\sigma} = -\frac{1}{2}(a^{\sigma} \circ b^{\sigma}) = -2(-\frac{1}{2}a^{\sigma}) \circ (-\frac{1}{2}b^{\sigma}) = -2(a^{\pi} \cdot b^{\pi}).$$

The universal property of $(U(J), \rho)$ implies that we have a homomorphism $\varphi: U(J) \to A(J)$. In view of fact, that $U(J)$ and $A(J)$ are generated by J^{ρ} and J^{σ}, φ is an epimorphism. This proves the lemma.

Theorem 2. The variety SN is not special.

Proof. If the variety SN is special, then a Cohn identity for (f, k^2) is valid in SN. By Lemma 7 the identity (9) is valid in the algebra $USJ_3[x, y, z]$. And hence, by Proposition 3, in the algebra A_3, which contradicts to the Lemma 8. This proves the theorem.

Therefore, we have proved the main result :

$$SpecN \underset{\neq}{\subset} SN \underset{\neq}{\subset} N ,$$

where :

(a) the algebra $A \in N$ and $A \notin SN$,

(b) the algebra $C \cong SJ_3[X]/K \in SN$ and $C \notin SpecN$.

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