GLA
(3)

Kac's announcement

A GLA is classical if it is simple and the representation of the even part on the odd part is completely reducible.

Example. Take $G_{-1} \oplus G_0 \oplus G_1$ with G_0 = matrices of trace 0, G_1 = symmetric matrices, G_{-1} = skew matrices. Get a simple Z-graded GLA by $[ab] = ab$, $[ca] = ca + ac^T$, $[cb] = -c^Tb - bc$ for a, b, c in G_1, G_{-1}, G_0. Call this P(m).

Prop. 2. A simple Z_2-graded GLA with nondegenerate Killing form is a suitable special linear or orthosymplectic algebra or a certain 40-dimensional algebra F(4).

Th. 1. The classical GLA's are as follows: all the special linear and orthosymplectic algebras, F(4), P(m), and the Gellman-Radicati algebras.

Now come the algebras of Cartan type. W(n) is the algebra of derivations of the Grassman algebra Λ on $x_1, \ldots, x_n$; its elements have the form $\sum P_i\partial_i$, $\partial_i(x_j) = \delta_{ij}$. Define two algebras Ω and S of differential forms on Λ. Ω(S) is the Λ-algebra generated by anticommuting (commuting) differentials $dx_1, \ldots, dx_n$ ($\delta x_1, \ldots, \delta x_n$); grade with $\deg dx_i = 1$, $\deg \delta x_i = 0$. Define on Ω(S) a differential $d(\delta)$ by $d(x_i) = dx_i$, $d^2(x_i) = 0$, and similarly for δ. Every derivation D of degree s on Λ induces a derivation on Ω(S) satisfying $Dd = dD$ ($D\delta = (-)^s\delta D$).

For $\omega \in \Omega$ or S set $L(\omega)$ = all $D \in W(n)$ with $D\omega = 0$. $S(n) = L(dx_1 \wedge \ldots \wedge dx_n)$; $\tilde{S}(n) = L((1 + x_1 \cdot \ldots \cdot x_n)dx_1 \wedge \ldots \wedge dx_n)$, $n = 2k$; $H(n) = L((\delta x_1)^2 + \ldots + (\delta x_n)^2)'$, ' denoting commuting algebra. The algebra S(n) is the linear span of the elements $(\partial_i\emptyset)\partial_j + (\partial_j\emptyset)\partial_i$, $\emptyset \in \Lambda$, and H(n) consists of the elements $(\partial_1\emptyset)\partial_1 + \ldots + (\partial_n\emptyset)\partial_n$, $\emptyset \in \Lambda$, where $\emptyset$ does not contain $x_1 \cdot \ldots \cdot x_n$. W(n), S(n), $\tilde{S}$(n), H(n) are simple for $n > 1, 2, 2, 3$ resp. Call these the GLA's of Cartan type.

Th. 2. Every Z-graded simple GLA $\sum\oplus G_i$ ($i \geq -1$) is SL, OSp(2r,1), P, W, S, or H.

Th. 3. Every Z_2-graded simple GLA is classical or of Cartan type.

It is easy to prove the theorem of the highest weight for representations of simple GLA's.

Th. 4. Let $G = G_0 \oplus G_1$ be a solvable GLA, and let there be given an irreducible non-one-dimensional representation of G on a graded vector space with even dimension m, odd dimension n. Then m = n = a power of 2.

The above is my crude, abridged translation of [7]. I have undoubtedly garbled parts of it.

There is only one other piece of news since the first newsletter. When Zumino visited here on Oct. 15 he told me about the following reference.

17. F. Berezin and M. Marinov, Classical spin and Grassman algebras, JETP Letters 21, 678-680 (1975) (Russian).