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My intention is to make this the final GLA newsletter. I theorize that this new specialty has now matured to the point where no further special efforts are needed to avoid duplication. I am also using this as an opportunity to announce the current state of my work on GJA's. The new bibliography lumps together new items, references previously incomplete, papers that were previously preprints, corrections, translations, etc.

36. G. R. Allcock, The intrinsic properties of rank and nullity of the Lagrange bracket in the one dimensional calculus of variations, Phil. Trans. Roy. Soc. Lon. 279(1975), 487-545.

37. ___, Invariant Lagrangian theory of the Poisson bracket for systems with constraints, Proc. Roy. Soc. Lon. A 344(1975), 175-198.

38. N. Backhouse, Some aspects of graded Lie algebras, 7 pp.

39. L. Corwin, Finite dimensional representations of semi-simple graded Lie algebras, 22 pp.

40. D. Ž. Djoković, Classification of some 2-graded Lie algebras, J. of Pure and Applied Alg. 7(1976), 217-230.

41. ___ and G. Hochschild, Semisimplicity of 2-graded Lie algebras II, Ill. J. of Math. 20(1976), 134-143.

42. B. Durand, Construction of two-sided Z-graded pseudo Lie algebras, 29 pp.

43. P. Freund and I. Kaplansky, Simple supersymmetries, J. Math. Phys. 17(1976), 228-231.

44. J. Hietarinta, Supersymmetry generators of arbitrary spin, Phys. Rev. D, 13(1976), 838-850.

45. G. P. Hochschild, Semisimplicity of 2-graded Lie algebras, Ill. J. of Math. 20(1976), 107-123.

46. V. G. Kac, Letter to the editor, Fn. Ana. and its Appl. 10(1976), no. 2, p. 93 (Russian). (This gives corrections to [7].)

47. ___, Classification of simple Lie superalgebras, Fn. Ana. and its Appl. (translation) 9(1975), 263-265. (This is a corrected translation of [7].)

48. ___, Characters of typical representations of classical Lie superalgebras, 6 pp.

49. D. A. Leites, Cohomologies of Lie superalgebras, Fn. Ana. and its Appl.(translation) 9(1975), 340-341. (This is a translation of [31].)

50. W. Nahm, V. Rittenberg, and M. Scheunert, The classification of graded Lie algebras, Phys. Letters 61B(1976), 383-384.

51. ___, Classification of all simple graded Lie algebras whose Lie algebra is reductive, I, 53 pp., II, 27 pp.

52. ___, Graded Lie algebras: Generalization of Hermitean representations, 27 pp.

53. ___, Irreducible representations of the osp(2,1) and spl(2,1) graded Lie algebras, 25 pp.

54. A. Pais and V. Rittenberg, Erratum: Semisiple graded Lie algebras, J. of Math. Phys. 17(1976), 598. (This is a correction to [23].)

55. L. E. Ross, Representations of graded Lie algebras, Trans. Amer. Math. Soc. 120(1965), 17-23. (In [13] this was previously listed as an unpublished thesis.)

56. H. Tilgner, A graded generaliztion of Lie triples, 8 pp.

57. ___, Z_2- graded generalizations of some classical Lie algebras and curvature structures, 15 pp.

GJA's

Let J be a graded algebra (graded mod 2). Call J Jordan if it is commutative graded style (ab = ba except that ab = -ba when a, b are both odd) and satisfies the appropriate fourth degree identity. The choice is motivated by the following fact. Consider an ungraded identity built out of product, Lie bracket, and Jordan bracket. Then this becomes a valid graded identity if a minus sign is placed on each term in which the odd variables undergo an odd permutation. (I announced this as a conjecture in lectures at St. Andrews (July, 1976) and jokingly called it the "infallible guide". But actually it is ridiculously easy to prove.) So we take the linearized Jordan identity

$$ab.cd + ac.db + ad.bc = (bc.a)d + (cd.a)b + (db.a)c$$

subject to the sign convention just mentioned.

A graded associative algebra under the graded Jordan bracket yields a "special" GJA. But there is a finer distinction to be made here, depending on representability within a finite-dimensional associative algebra. The distinction really exists (in other words, the analogue of Ado's theorem fails).

The list of simple GJA's (over an algebraically closed field of characteristic 0) is as follows: 6 infinite families and 2 "sporadic" ones. The families: full linear, orthosymplectic, analogue of the Gell-Mann-Michel-Radicati GLA, analogue of Kac's P(m), graded version of degree 2 Jordan algebras, and a family of 4-dimensional GJA's. (As McCrimmon pointed out, some of these 4-dimensional GJA's should have appeared in my GJA preprint.) The 3-dimensional one has basis e, a, b (e even, a and b odd) with $e^2 = e$, ea = a/2, eb = b/2, ab = e. It has no unit element. The 10-dimensional one admits no associative representation (even infinite-dimensional). But I don't know whether it is a homomorphic image of a special GJA. Several random substitutions into the graded versions of Glennie's identities were inconclusive.

Nil GJA's are solvable but not necessarily nilpotent. The obvious choice for the radical is the maximal solvable ideal, and semisimplicity means that it vanishes. Semisimple GJA's need not be direct sums of simple ones: just adjoin a unit element to the 3-dimensional one above. But all is not lost. It is possible to classify the semisimple indecomposable ones: each is either simple or the result of adjoinig a unit element to a direct sum of copies of the 3-dimensional algebra.

The first step is the proof that the even part of a semisimple GJA is semisimple. I prove this by juggling the Peirce decomposition and using the fact that a derivation of an ungraded Jordan algebra sends the radical into itself. (I temporarily thought that this easy theorem was not explicitly in the literature, but Schafer pointed to p. 869 of Jacobson's paper in vol. 50 of the Annals, and McCrimmon mentioned p. 1071 of Jacobson's Albert obituary.) This is the first of several critical uses of characteristic 0. Nevertheless, it remains possible that the GJA results hold for characteristic p. I have postponed writing a detailed account, hoping to settle this.

On June 21, 1976 I received from Kac a letter dated June 9, 1976 in which he announced the classification of simple GJA's (also over an algebraically closed field of characteristic 0). The results agree except for his omission of the three-dimensional algebra (an omission perhaps connected with its lack of a unit element).