Newsletter no. 1 on GLA's (graded Lie algebras) Oct. 13, 1975

I. K., Chicago

I am suddenly inspired to try to serve as a clearing house for the current flurry of activity on GLA's. I am sending this newsletter to every mathematician I believe to be interested and to several physicists. Below is a stab at a bibliography. I take [1] as supplying the basic bibliography. I have added all mathematical items known to me that do not appear in [1]. Some additional physics titles listed in [5] are not reproduced here. Finally I list the preprints that have reached me.

I'll throw in a bit of history. Maybe others will feel in the mood too. Freund contacted Herstein around the Spring of 1975. They brought me in on it in May. A seminar started up in June, adjourned for the summer, and resumed in October. [5] was drafted in June, revised in September. When the first draft was written we knew about the "special linear" ones, the "orthosymplectic" ones, and (vaguely) additional ones. The 17-dimensional $\Gamma(A,B,C)$ popped up a week or so later. Letters in July to Freund in Colorado announced the 31-dimensional Γ_2 and the 40-dimensional Γ_3. [8] was completed in early August; most of the work was done while I was a guest of UCLA. The two drafts of [9] were written in early September. Then I heard of the work of Kac in September and that of Hochschild and Djoković in October.

Comments. [9] is incomplete in that Γ_3 is not constructed. Otherwise, existence and uniqueness is proved for all algebras going with the root systems of [8]. Another unanswered question is whether the form on $\Gamma(A,B,C)$ comes from a representation. A 13-page first draft of [9] contains some errors. I have work under way on representations, derivations (i. e. 1-cohomology), automorphisms, and real forms.

I have not seen [7]. As I understand it, it announces a complete classification of simple GLA's but omits Γ_2. Kac's notation for Γ_3 is F(4) and he suggests G(3) for Γ_2.

Hochschild wrote me that Ado's theorem for GLA's is proved in [13]. Kostant sent me a proof of Ado's theorem in an undated letter (Sept. 1975). Kostant also described ~~briefly~~ his current work on representations, using his "orbit method".

In [2] the simple GLA's with simple even part are classified. There are 4 infinite families. The first, in the notation of [5], is the orthosymplectic family OSp(2r|1). Moreover in [3] it is shown that the algebras of this family are the only simple GLA's for which every representation is completely reducible. The second infinite family is that of Gell-Mann and Radicati (see p. 567 of [1]). The third and fourth have as even part all linear transformations of trace 0 on a vector space V; the odd part is resp. $S^2V + (\Lambda^2V)^*$, $(S^2V)^* + \Lambda^2V$. Here S is symmetric product, Λ is exterior product, and * is dual. Only the products between S and Λ are nonzero; this product is defined in [3].

1. L. Corwin, Y. Ne'eman, and S. Sternberg, Graded Lie algebras in mathematics and physics (Bose-Fermi symmetry), Reviews of Modern Physics 47(1975), 573-604.

2. D. Djoković, Classification of some 2-graded Lie algebras, 25 pp.

3. ____ and G. Hochschild, Semisimplicity of 2-graded Lie algebras II, 19pp.

4. P. Freund, Conformal algebra in superspace and supergauge theory, 9 pp.

5. ____ and I. Kaplansky, Simple supersymmetries, 14 pp., submitted to the Jour. of Math. Physics

6. G. Hochschild, Semisimplicity of 2-graded Lie algebras, 31 pp.

7. V. G. Kac, Classification of simple Lie superalgebras, Functional Analysis and its Applications 9(1975), 91-92 (Russian).

8. I. Kaplansky, Graded Lie algebras I, 59pp.

9. ____, Graded Lie algebras II, 17pp., incomplete second draft.

10. J. P. May, The cohomology of restricted Lie algebras and of Hopf algebras, Bull. Amer. Math. Soc. 71(1965), 372-377.

11. ____, Same title, J. of Alg. 3(1966), 123-146.

12. J. W. Milnor and J. C. Moore, On the structure of Hopf algebras, Ann. of Math. 81(1965), 211-264.

13. Leonard E. Ross, 1964 Berkeley Ph. D. thesis (unpublished).

14. R. Speers, Lie structures in simple graded rings, Duke Math. J. 38(1971), 81-92.

15. S. Sternberg and J. Wolf, Graded Lie algebras and bounded homogeneous domains, 16 pp.

16. ____, Charge conjugation and Segal's cosmology, 34 pp.

D. Ž. Djoković
U. of Waterloo
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