ERRATUM TO "ON δ-DERIVATIONS OF LIE ALGEBRAS AND SUPERALGEBRAS"

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In [Z], an incorrect definition of the Grassmann envelope of a Lie superalgebra was used. Namely, if $L = L_0 \oplus L_1$ is a Lie superalgebra, and $G = G_0 \oplus G_1$ is the Grassmann algebra, the multiplication on $G(L) = (L_0 \otimes G_0) \oplus (L_1 \otimes G_1)$ can be defined in two ways; as for ordinary algebras:

$$[x \otimes g, y \otimes h] = [x, y] \otimes gh,$$

and as for superalgebras:

$$[x \otimes g, y \otimes h] = (-1)^{\deg(g)\deg(y)}[x, y] \otimes gh,$$

where x, y and g, h are homogeneous elements of L and G, respectively. These two multiplications lead to what is called in [B, p. 72] the Grassmann envelopes of the first and second kind. However, as justly remarked in the footnote at the same page of [B], the first definition contradicts the Sign Rule, and will lead, sooner or later, to confusion. [Z] is an instance of such a confusion: it is easy to see that the Grassmann envelope of the first kind is, generally, not a Lie algebra, what defeats its main utility, at least in the context of [Z].

To remedy this, one should adopt the definition of the Grassmann envelope of the second kind, and inject signs at appropriate places. First, the map in Lemma 4.1(ii) should be defined as

$$a_0 \otimes b_0 + a_1 \otimes b_1 \mapsto D(a_0) \otimes \chi(b_0) + (-1)^{\deg(D)} D(a_1) \otimes \chi(b_1).$$

Second, the symmetric bilinear form used in the proof of Theorem 4.8 should be defined as

$$(x \otimes g, x' \otimes g') = (-1)^{\deg(g)\deg(x')}(x, x')f(gg').$$

No modifications in the proofs are needed.

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References

[B] F. Berezin, *Introduction to superanalysis*, 2nd ed., revised and edited by D. Leites, with appendix *Seminar on Supersymmetry. Vol. $1\frac{1}{2}$*, MCCME, Moscow, 2013 (in Russian); D. Reidel, 1987 (English translation of the 1st ed.).

[Z] P. Zusmanovich, *On δ-derivations of Lie algebras and superalgebras*, J. Algebra **324** (2010), 3470–3486; arXiv:0907.2034.

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