

ERRATUM TO “A CONVERSE TO THE SECOND WHITEHEAD LEMMA”

PASHA ZUSMANOVICH

In [Z], a result from [R] is used, claiming that the Hochschild–Serre spectral sequence abutting to the cohomology of the semidirect sum $L = S + I$ of Lie algebras S and I (S is a subalgebra, I is an ideal, i.e. S acts on I) with coefficients in an arbitrary L -module V , with respect to the ideal I , stabilizes at the E_2 page ([R, Lemma 1] and [Z, Proposition 1.3]). In the whole generality, this statement is false. In fact, as shown in [B], this spectral sequence can stabilize at arbitrarily large step.

This statement is true, however, in many special cases: for example, if the base field is of characteristic zero, S is a finite-dimensional semisimple Lie algebra, and V is finite-dimensional ([HS, Theorem 13]). More sufficient conditions guaranteeing stabilization at E_2 can be found in [B, Theorem 3], [DP1, Corollaries 1.5, 1.6 and Theorem 1.7], and [DP2, Theorem 1.2].

The main result (Theorem 0.2) of [Z] can be salvaged in the following way. The above-mentioned general erroneous claim was used once, in the proof of Lemma 2.1. Lemma 2.1, in its turn, was used in the proof of Theorem 0.2 twice: first time (in the proof of Lemma 2.4) in the situation where S is semisimple, what is covered by the Hochschild–Serre result mentioned above, and the second time in the situation where $L = S + I$ is the direct sum of algebras. In the latter case, one can invoke the Künneth theorem instead of Proposition 1.3.

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INSTITUTE OF MATHEMATICS AND STATISTICS, UNIVERSITY OF SÃO PAULO, BRAZIL
Email address, as of April 20, 2024: pasha.zusmanovich@gmail.com