MR2760365 (Review) 16E40 (16E05)

Generalov, A. I. (RS-STPT)

The Hochschild cohomology of quaternion-type algebras. III. Algebras with a small parameter. (Russian. English, Russian summaries)

Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **356** (2008), *Voprosy Teorii Predstavleniĭ Algebr i Grupp*. 17, 46–84, 189; *translation in J. Math. Sci. (N. Y.)* **156** (2009), *no. 6,* 877–900.

{This review is also provided for [A. I. Generalov, A. A. Ivanov and S. O. Ivano, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **349** (2007), Voprosy Teorii Predstavleniĭ Algebr i Grupp. 16, 53–134, 243; MR2742854].}

In these two companion papers, the graded algebra structure of the Hochschild cohomology of a certain class of algebras $Q(2\mathcal{B})_1$ over a field of characteristic 2 with coefficients in itself is described explicitly.

The algebras from the class $Q(2\mathcal{B})_1$ are given by generators and relations depending on 4 parameters, 2 integers and 2 elements of the ground field.

The proofs are very computationally intensive.

Reviewed by *Pasha Zusmanovich*

MR2455517 (2009j:17013) 17B50

Premet, Alexander (4-MANC-SM); **Strade, Helmut** (D-HAMB)

Simple Lie algebras of small characteristic. VI. Completion of the classification. (English summary)

J. Algebra **320** (2008), *no. 9,* 3559–3604.

The celebrated Kostrikin-Shafarevich conjecture states that each finite-dimensional simple Lie algebra over an algebraically closed field of positive characteristic p is either of classical or Cartan type. During the last half-century, a lot of people contributed to different aspects of this classification.

The previous highlights include the classification of simple restricted Lie algebras by R. E. Block and R. L. Wilson [J. Algebra **114** (1988), no. 1, 115–259; MR0931904 (89e:17014)] and classification of simple (not necessarily restricted) Lie algebras of characteristic $p > 7$ by the second author [Trans. Amer. Math. Soc. **350** (1998), no. 7, 2553–2628; MR1390047 (98j:17020)].

In the series of papers of which the present one is the last, the authors rework the classification to cover the case $p > 3$. As was discovered by Melikyan, in $p = 5$ there is a new class of simple Lie algebras neither of classical nor of Cartan type, so this classification requires a treatment of new phenomena not occurring in larger characteristics.

And indeed, this last paper is devoted to thorough analysis of Melikyan algebras and other situations peculiar to the case $p = 5$. Namely, it is proved that if, for a simple Lie algebra L, the p-envelope of $\operatorname{ad} L$ in $\operatorname{Der} L$ contains a torus of maximal dimension whose centralizer in $\operatorname{ad} L$ acts non-triangulably on L, then $p = 5$ and L is isomorphic to a Melikyan algebra.

Together with results from the previous paper in the series [Part V, A. A. Premet and H. Strade, J. Algebra **314** (2007), no. 2, 664–692; MR2344582 (2008j:17039)], this completes the classification: each finite-dimensional simple Lie algebra over an algebraically closed field of characteristic $p > 3$ is either of classical type, Cartan type, or is a Melikyan algebra.

The way of the proof is typical for classification results: one carefully analyses all possible 2-sections in a root space decomposition with respect to a suitably chosen torus, to produce a subalgebra of codimension 5 with specific properties.

In the course of the proof results about Melikyan algebras are obtained, interesting for their own sake: it is proved that the simplest algebra in the series, $M(1,1)$, has no nontrivial central extensions, its irreducible modules of low dimension are described, and it is also proved that all Melikyan algebras possess a nonzero symmetric bilinear invariant form.

Some other auxiliary results require an analysis of the Hamiltonian Lie algebra $H(2,(2,1))$; in particular, conjugacy classes of toral elements of this algebra are described.

The reviewer could not resist the, perhaps superficial, temptation to compare this classification with the classification of finite simple groups: both efforts spanned decades; involved a lot of work of many people scattered over numerous journal papers, some of them of a very big volume; often require a very careful analysis of a huge number of cases; and "second generation" and "third generation" classification efforts emerged. However, the classification of simple Lie algebras, being arguably a simpler task than classification of simple groups, is in much better shape: it is smaller in volume, and the situation seems to be under the full control of a few experts.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2411970 (2009b:17021) 17B20 (17B01 17B70)

Koryukin, A. N. (RS-AOSSI)

Gröbner-Shirshov bases of the Lie algebra B_n^+. (Russian. Russian summary)

Algebra i Analiz **20** (2008), *no. 1,* 93–137; *translation in St. Petersburg Math. J.* **20** (2009), *no. 1,* 65–94.

In the remarkable paper [L. A. Bokut and A. A. Klein, Internat. J. Algebra Comput. **6** (1996), no. 4, 389–400, 401–412; MR1414346 (97k:17005)], two classical algebraic topics—finite-dimensional simple split Lie algebras of characteristic zero and the machinery of Gröbner-Shirshov bases—were blended: namely, the Serre relations among the standard generators of a classical simple Lie algebra were extended to the Gröbner-Shirshov basis of that algebra. Since then, the pupils of Bokut have extended and developed this blend of topics.

In this paper, the Gröbner-Shirshov basis is constructed for generators of the positive part B_n^+ (= nilradical of the Borel subalgebra) of a simple classical Lie algebra of type B_n. The novelty of the work is that arbitrary orders of generators corresponding to simple roots are considered (for a fixed order this follows already from the work of Bokut and Klein).

The amount of combinatorial computations is tremendous, but at the end the author not only exhibits the desired basis in each of the $n!$ cases in terms of certain graphs associated with the corresponding root system, but arrives at the following interesting result: If a graded Lie algebra is given by homogeneous relations such that each homogeneous component is nonzero if and only if the corresponding homogeneous component for B_n^+ is nonzero, then the algebra is isomorphic to B_n^+.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2742854 (Review) 16E40 (16E05)

Generalov, A. I. (RS-STPT); **Ivanov, A. A.** (RS-STPT);

Ivanov, S. O. [Ivano, Sergeĭ Olegovich] (RS-STPT)

The Hochschild cohomology of quaternion-type algebras. II. The series $Q(2\mathcal{B})_1$ in characteristic 2. (Russian. English, Russian summaries)

Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **349** (2007), *Voprosy Teorii Predstavleniĭ Algebr i Grupp*. 16, 53–134, 243; *translation in J. Math. Sci. (N. Y.)* **151** (2008), *no. 3,* 2961–3009.

{This review is also provided for [A. I. Generalov, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **356** (2008), Voprosy Teorii Predstavleniĭ Algebr i Grupp. 17, 46–84, 189; MR2760365].}

In these two companion papers, the graded algebra structure of the Hochschild cohomology of a certain class of algebras $Q(2\mathcal{B})_1$ over a field of characteristic 2 with coefficients in itself is described explicitly.

The algebras from the class $Q(2\mathcal{B})_1$ are given by generators and relations depending on 4 parameters, 2 integers and 2 elements of the ground field.

The proofs are very computationally intensive.

Reviewed by *Pasha Zusmanovich*

MR2469415 (2010h:16024) 16E40 (16D50 16E05 16G60 16G70)

Volkov, Yu. V. (RS-STPT); **Generalov, A. I.** (RS-STPT)

Hochschild cohomology for self-injective algebras of tree type D_n. I.
(Russian. English, Russian summaries)

Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **343** (2007), *Vopr. Teor. Predts. Algebr. i Grupp. 15,* 121–182, 273–274; *translation in J. Math. Sci. (N. Y.)* **147** (2007), *no. 5,* 7042–7073.

It is known that finite-dimensional associative self-injective algebras of finite representation type over an algebraically closed field are described in terms of a tree associated with a Dynkin diagram of type A, D or E. Here the authors construct a projective resolution of such algebras in the case of Dynkin diagrams of type D, and compute dimensions of the Hochschild cohomology of an algebra with coefficients in itself. The proofs are very computationally intensive, with many case-by-case considerations.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2389419 (2009a:16013) 16E40 (16S99)

Dolguntseva, I. A. (RS-AOSSI)

The Hochschild cohomology for associative conformal algebras. (Russian. Russian summary)

Algebra Logika **46** (2007), *no. 6,* 688–706, 792; *translation in* *Algebra Logic* **46** (2007), *no. 6,* 373–384.

In this paper, a variant of cohomology theory for conformal associative algebras is suggested. It is shown that extensions of an associative conformal algebra by a bimodule are described by the second cohomology, exactly as in the case of ordinary associative algebras and the Hochschild cohomology. It is shown that the second cohomology of a conformal Weyl algebra with coefficients in any bimodule is trivial. An example of a subalgebra of a conformal Weyl algebra with a nontrivial second cohomology is given.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2366357 (2008m:17013) 17B10 (16E10 17B55)

Mazorchuk, Volodymyr (S-UPPS)

Some homological properties of the category $\mathcal{O}$. (English summary)

Pacific J. Math. **232** (2007), *no. 2,* 313–341.

The category $\mathcal{O}$ in the title of the paper refers, as usual, to the celebrated Bernstein-Gel′fand-Gel′fand category of representations of a complex semisimple Lie algebra. In this paper:

- a description of homological dimension of various important classes of modules (such as projective, injective, simple, Verma, indecomposable tilting, etc.) in the principal block $\mathcal{O}_0$ (= decomposable direct summand containing the trivial module) of $\mathcal{O}$ is given;
- results on extensions between Verma modules are obtained. The extension algebra is considered as a $\mathbb{Z}^2$-graded object and it is shown that in some cases it is Koszul. Ext^1's between some classes of modules in $\mathcal{O}_0$ are determined explicitly;
- it is shown that some classes of modules have linear projective resolution, either in the category $\mathcal{O}$ itself, or in the Ringel dual of $\mathcal{O}_0$.

Reviewed by *Pasha Zusmanovich*

MR2344582 (2008j:17039) 17B50 (17B20)

Premet, Alexander (4-MANC-SM); **Strade, Helmut** (D-HAMB)

Simple Lie algebras of small characteristic. V. The non-Melikian case. (English summary)

J. Algebra **314** (2007), *no. 2,* 664–692.

This paper, the penultimate in the series, is part of a tour de force to revise and extend the earlier classification of finite-dimensional simple Lie algebras over an algebraically closed field of positive characteristic and to cover the cases of low characteristic.

Recall that a torus in a Lie algebra is called standard if the commutant of the centralizer of the torus consists of elements acting nilpotently on the whole algebra. The main result of the paper says that if, in a finite-dimensional simple Lie algebra over an algebraically closed field of characteristic $p > 3$, all tori of maximal dimension in its p-envelope are standard, then the algebra is either of classical or of Cartan type.

If $p > 5$, this settles the classification entirely. If $p > 3$, this leaves open the case when $p = 5$ and the p-envelope of the algebra contains nonstandard torus of maximal dimension. The latter case involves the Melikyan algebras and is settled in the last paper in the series [A. A. Premet and H. Strade, "Simple Lie algebras of small characteristic. VI. Completion of the classification", preprint, arxiv.org/abs/0711.2899].

The technique is typical for classification papers and consists of a careful consideration of properties of roots and sections with respect to suitable torus, and passing to an associated graded algebra. By a careful analysis, the authors slightly modify the reasoning of [H. Strade, Ann. of Math. (2) **130** (1989), no. 3, 643–677; MR1025169 (91a:17023)] and [G. M. Benkart, J. M. Osborn and H. Strade, Trans. Amer. Math. Soc. **341** (1994), no. 1, 227–252; MR1129435 (94c:17035)], to cover the case $p > 3$.

{For Part IV see [A. A. Premet and H. Strade, J. Algebra **278** (2004), no. 2, 766–833; MR2071664 (2005e:17032)].}

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2301242 (2008d:17015) 17B35 (11B68 14L05 14L15 16S80 16W30)

Durov, Nikolai (D-MPI); **Meljanac, Stjepan** (CT-IRBO-TPD);
Samsarov, Andjelo (CT-IRBO-TPD); **Škoda, Zoran** (CT-IRBO-TPD)

A universal formula for representing Lie algebra generators as formal power series with coefficients in the Weyl algebra. (English summary)

J. Algebra **309** (2007), *no. 1,* 318–359.

The authors provide a constructive way to represent an n-dimensional Lie algebra of characteristic 0 via formal power series with coefficients in the Weyl algebra A_n. This can also be viewed as the realization of the universal enveloping algebra as a deformation of the polynomial algebra inside the Weyl algebra. Bernoulli numbers are involved.

Three different proofs are given: first, direct quite involved computations with tensors; second, using computations with formal vector fields on a formal group (for this, a suitable piece of Lie theory based on formal group schemes is developed); and third, using coderivations and Hopf algebras.

The authors outline further interesting possible developments: to put all this into an operadic framework, and to generalize to Leibniz algebras and quantum groups.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2260226 (2007k:15045) 15A69 (15A21)

Belitskii, Genrich R. (IL-BGUN); **Sergeichuk, Vladimir V.** (UKR-AOS)

Congruence of multilinear forms. (English summary)

Linear Algebra Appl. **418** (2006), *no. 2-3,* 751–762.

Two multilinear forms $F: U\times\cdots\times U\to K$ and $G: V\times\cdots\times V\to K$ are called symmetrically equivalent if there exist linear bijections $\varphi_1,\dots,\varphi_n: U\to V$ such that $F(u_1,\dots,u_n)=G(\varphi_{\sigma(1)}(u_1),\dots,\varphi_{\sigma(n)}(u_n))$ for any permutation σ on n letters. The forms are called congruent if all φ_i's can be chosen equal.

The authors prove that in the case when the underlying vector spaces U and V are finite-dimensional and the ground field K is the field of complex numbers, symmetric equivalence implies congruence. This is similar to the known result for matrices. It is interesting whether this remains true over an arbitrary algebraically closed ground field of characteristic zero. The proof given in the paper utilizes Taylor series expansion, so it is peculiar to the complex case.

Over the field of real numbers the relationship between symmetric equivalence and congruence is also given, and it is, as expected, more subtle.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2253565 (2007d:16018) 16E40

Bezyakina, E. A. (RS-STPT); **Generalov, A. I.** (RS-STPT)

Cocycles in the relative Hochschild cohomology. (Russian. English, Russian summaries)

Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **330** (2006), *Vopr. Teor. Predst. Algebr. i Grupp. 13,* 29–35, 271; *translation in J. Math. Sci. (N. Y.)* **140** (2007), *no. 5,* 622–625.

This paper belongs to the area of classical homological algebra à la Cartan-Eilenberg, which, perhaps surprisingly, allows new discoveries even today.

The authors are concerned with a canonical isomorphism between the extension module $\mathrm{Ext}^n_A(M,N)$ and the Hochschild cohomology $H^n(A,\mathrm{Hom}(M,N))$ for the right modules M, N over an associative algebra A.

Earlier V. B. Dlab and C. M. Ringel [Tsukuba J. Math. **14** (1990), no. 2, 489–496; MR1085213 (92d:16018)] constructed an explicit such isomorphism. Here the authors present a relative version of this construction, in which both extension and cohomology are taken relative to a certain subalgebra of A. They also note one inaccuracy in [op. cit.].

Reviewed by *Pasha Zusmanovich* (Reykjavík)

MR2249129 (2007f:16020) 16E45 (16E40)

Ladoshkin, M. V. (RS-MORD)

A_∞-modules over A_∞-algebras and the Hochschild cohomology complex for modules over algebras. (Russian. Russian summary)

Mat. Zametki **79** (2006), *no. 5,* 717–728; *translation in Math. Notes* **79** (2006), *no. 5-6,* 664–674.

There is a known relationship between Hochschild cohomology of a differential graded algebra and a set of all A_∞-algebra structures on that algebra [see V. A. Smirnov, Izv. Ross. Akad. Nauk Ser. Mat. **64** (2000), no. 5, 147–162; MR1789189 (2001i:18017); and references therein].

In this paper, this relationship is generalized to the case of differential graded modules over differential graded algebras. Namely, for a given dg-module over a given dg-algebra, a Hochschild-like cochain complex is constructed whose cohomology is responsible for the existence of structures of an A_∞-module over an A_∞-algebra. In particular, the vanishing of cohomology of this complex in certain dimensions implies the existence of a unique, up to isomorphism, A_∞-module structure.

Reviewed by *Pasha Zusmanovich* (Reykjavík)

MR2238902 (2007d:17029) 17B56 (17B68 17B69)

Linshaw, Andrew R. (1-BRND)

The cohomology algebra of the semi-infinite Weil complex. (English summary)

Comm. Math. Phys. **267** (2006), *no. 1,* 13–23.

The author describes both the Lie superalgebra structure and associative algebra structure of the cohomology of the semi-infinite Weil complex associated to the Virasoro algebra, which coincides with the BRST complex of a conformal vertex algebra with central charge 26.

The same cohomology as a vector space was earlier computed by B. L. Feĭgin and E. V. Frenkel [Comm. Math. Phys. **137** (1991), no. 3, 617–639; MR1105434 (92h:17028)].

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2203713 (2006j:17005) 17A36

Moreno, Guillermo [Moreno R., Guillermo] (MEX-IPN-CI)

Monomorphisms between Cayley-Dickson algebras. (English summary)

Non-associative algebra and its applications, 281–289, *Lect. Notes Pure Appl. Math.*, 246, *Chapman & Hall/CRC, Boca Raton, FL*, 2006.

The author studies monomorphisms between the famous Cayley-Dickson algebras over the field of real numbers (obtained via a doubling process starting from a one-dimensional unital algebra, whose initial terms are complex numbers, quaternions and octonions).

He determines all monomorphisms from the algebras of complex numbers and quaternions into any Cayley-Dickson algebra and obtains partial results about monomorphisms from octonions into any Cayley-Dickson algebra. This agrees with the well-known results about automorphisms of the respective algebras.

The language, typesetting, and references of the article are problematic. Some definitions are repeated twice, while some are missing and must be guessed from the context.

The method consists of elementary calculations, in which the reviewer has not found any mistakes.

{For the entire collection see MR2203689 (2006h:17002)}

Reviewed by *Pasha Zusmanovich*

MR2199629 (2006m:16010) 16E45 (16E05 53D17 53D55)

Dolgushev, Vasiliy [Dolgushev, V. A.] (RS-ITEP)

A formality theorem for Hochschild chains. (English summary)

Adv. Math. **200** (2006), *no. 1,* 51–101.

The author proves the existence of a quasi-isomorphism of two differential graded modules over respective differential graded algebras: exterior forms on a smooth real manifold M over polyvector fields on M and Hochschild chains on algebra $C^\infty(M)$ over polydifferential operators on M.

This lies at an intersection of two (related) themes: first, the celebrated Hochschild-Kostant-Rosenberg theorem and its generalizations (claiming quasi-isomorphism of the respective pairs of modules and algebras separately, without consideration of the module structures), and, second, formality theorems and conjectures of Kontsevich, Tsygan, Shoikhet and others.

The main tool is a type of Fedosov resolutions of the respective modules and algebras.

As an application, the author computes the Hochschild homology of the quantum algebra of functions on a Poisson manifold.

This is a clear exposition of a technically difficult subject.

Reviewed by *Pasha Zusmanovich*

MR2189869 (2006m:17019) 17B55 (16E40 16S32 17B56 19C09)

Lau, Michael [Lau, Michael Kekainalu] (3-OTTW-MS)

On universal central extensions of $\mathfrak{sl}_n(A)$. (English summary)

Infinite-dimensional aspects of representation theory and applications, 43–54, *Contemp. Math.*, 392, *Amer. Math. Soc., Providence, RI*, 2005.

It is well known that the classical (co)homology of a matrix Lie algebra over an associative algebra A is intimately related to the cyclic (co)homology of A. In particular, central extensions of such Lie algebras are described by (and, in the generic case, coincide with) the first-order cyclic cohomology of A (first observed by C. Kassel and J.-L. Loday [Ann. Inst. Fourier (Grenoble) **32** (1982), no. 4, 119–142 (1983); MR0694130 (85g:17004)]).

Here the author applies this machinery to the situation when $A = A_{r,s}$, a localization of the Weyl algebra A_r. By the general result mentioned above, this amounts to computation of the first-order cyclic cohomology of $A_{r,s}$.

The reviewer does not understand a remark in the last paragraph of the paper about "applicability" of this method in the case of a non-Lie-perfect algebra A: it is quite obvious that any "non-perfectness" could be dealt with in an elementary, but a bit tedious way (like, for example, in the reviewer's paper [Astérisque No. 226 (1994), 11, 435–452; MR1317128 (96a:17015)]). However, in that questionable context the author poses a conjecture which has an independent interest: any ring of differential operators on any nonsingular affine variety is Lie-perfect in characteristic 0.

{For the entire collection see MR2188504 (2006g:00014)}

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2183970 (2006h:17032) 17B68 (17B40)

Lin, Weiqiang (PRC-XIAM); **Tan, Shaobin** (PRC-XIAM)

Central extensions and derivations of the Lie algebras of skew derivations for the quantum torus. (English summary)

Comm. Algebra **33** (2005), *no. 11,* 3919–3938.

The authors determine central extensions and derivations of a certain family of Z_2-graded Virasoro-like complex Lie algebras, parametrized by a complex number q.

The cases when $q = 1$ and when q is not a root of unity were studied by E. E. Kirkman, C. Procesi and L. W. Small [Comm. Algebra **22** (1994), no. 10, 3755–3774; MR1280096 (96b:17016)]. Here the authors study the remaining, more complicated, case when q is a root of unity, $q \neq 1$.

Reviewed by *Pasha Zusmanovich*

MR2161515 (2006e:16048) 16S37 (16E05 16S15)
Piontkovskiĭ, D. I. (RS-AOS-B)
Koszul algebras and their ideals. (Russian. Russian summary)
Funktsional. Anal. i Prilozhen. **39** (2005), *no. 2,* 47–60, 95; *translation in Funct. Anal. Appl.* **39** (2005), *no. 2,* 120–130.

The notion of Koszul filtration and Koszul flags of commutative associative algebras was introduced by A. Conca [see, e.g., Math. Ann. **317** (2000), no. 2, 329–346; MR1764242 (2001c:13020)]. A Koszul flag is a finite chain of ideals of an algebra, generated by linear forms and having a linear free resolution, and a Koszul filtration is a certain technical condition on a set of ideals which ensures that it contains a Koszul flag.

The goal of this paper is to consider the noncommutative situation. The author studies related notions, such as Koszul algebras, initially Koszul algebras (those which admit a minimal Koszul filtration), algebras admitting Koszul filtration, PBW-algebras, relationships between these classes of algebras, and provides examples and counterexamples.

It is proved that an algebra with a Koszul filtration has a rational Hilbert series. Necessary and sufficient conditions are given for a linear flag to be Koszul and for an algebra to be initially Koszul (the latter in terms of Gröbner bases).

The paper is concluded with a list of open questions.

Reviewed by *Pasha Zusmanovich* (Reykjavík)

MR2150849 (2006e:17023) 17B40 (17B68)
Zha, Jianguo (PRC-TONG-AM); **Hu, Jianhua [Hu, Jian Hua[1]]** (PRC-TONG-AM)
Certain classes of automorphisms of infinite rank affine Lie algebras. (English summary)
Comm. Algebra **33** (2005), *no. 6,* 1893–1901.

The authors consider affine Lie algebras associated with infinite Cartan matrices (infinite rank affine algebras). While the automorphisms of finite rank affine Lie algebras have been described [D. H. Peterson and V. G. Kac, Proc. Nat. Acad. Sci. U.S.A. **80** (1983), no. 6 i., 1778–1782; MR0699439 (84g:17017)], the similar question for infinite rank is open.

Each infinite rank affine algebra may be represented as an inductive limit of a chain of finite-dimensional simple Lie algebras. The authors determine the group of automorphisms of an affine

Lie algebra leaving invariant any given infinite subset in this chain.

The description is very similar to the description of the automorphism groups of classical simple Lie algebras: the group is generated by 3 types of automorphisms—inner, diagonal, and diagram automorphisms.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2131013 (2005m:16014) 16E45 (16S32 18D50 55S20)

Tamarkin, Dmitri [Tamarkin, Dmitry E.] (1-NW); **Tsygan, Boris** (1-NW)

The ring of differential operators on forms in noncommutative calculus.

Graphs and patterns in mathematics and theoretical physics, 105–131, *Proc. Sympos. Pure Math.*, 73, *Amer. Math. Soc.*, *Providence, RI*, 2005.

The famous Hochschild-Kostant-Rosenberg theorem states the quasi-isomorphism of complexes $C^\bullet(C^\infty(M))$, the Hochschild cochain complex of the algebra $C^\infty(M)$ of smooth functions on a smooth manifold M, and $\mathcal{V}^\bullet(M)$, the algebra of multivector fields on M.

In ["Another proof of M. Kontsevich formality theorem for $\mathbb{R}^n$", preprint, arxiv.org/abs/math/9803025], the first author gave a noncommutative version of this theorem in the framework of so-called G_∞-algebras which generalizes notions of the L_∞-algebra and the Gerstenhaber algebra. He established, for an associative algebra A, a quasi-isomorphism of differential graded Lie algebras $C^\bullet(A)$ and $\mathcal{V}^\bullet(A)$, inducing an isomorphism of corresponding cohomology Gerstenhaber algebras. The few equivalent constructions of the Gerstenhaber algebra $\mathcal{V}^\bullet(A)$, known in the literature, were given either in an inexplicit or noncanonical ad hoc manner.

In this paper, the authors provide a canonical construction shedding more light on $\mathcal{V}^\bullet(A)$. Namely, they introduce a notion of an enveloping algebra $Y(A^\bullet)$ of a Gerstenhaber algebra $A^\bullet$ and establish a quasi-isomorphism of algebra $Y(\mathcal{V}^\bullet(A))$ and a very interesting object—the algebra of Hochschild chains of the differential graded associative algebra $C^\bullet(A)$. In the venerable case $A^\bullet = \mathcal{V}^\bullet(M)$, $Y^\bullet(A)$ coincides with the algebra of differential operators on differential forms on M.

Proofs are only sketched. The discussion of ideas and notions, however, is very detailed, elucidating complex relationships in the monstrous world of Gerstenhaber algebras, L_∞-algebras, A_∞-algebras and similar structures with a hierarchy of higher operations.

{For the entire collection see MR2131006 (2005j:00017)}

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2102076 (2005h:17057) 17B70 (17B30)

Shumyatsky, Pavel (BR-BRSL); **Tamarozzi, Antonio**; **Wilson, Lawrence** (1-FL)

Z_n**-graded Lie rings. (English summary)**

J. Algebra **283** (2005), *no. 1,* 149–160.

The authors improve the previously known bounds for the nilpotency class and derived length of Z_n-graded Lie rings with a zero null component: the derived length of such a ring is at most $2^{n-4} + \lfloor \log_2(n-1) \rfloor$ (if n is a prime, a slightly better bound is achieved), and the nilpotency class of a Z_p-graded Lie ring L with $L^{(s)} = 0$ is at most $\frac{(p-2)^s - 1}{p-3}$.

The authors note that these bounds are probably still very far from the best possible ones.

The arguments consist of a repeated skillful application of the Jacobi identity.

Reviewed by *Pasha Zusmanovich*

MR2120204 (2006e:16018) 16E40

Zharinov, V. V. (RS-AOS)

Hochschild cohomology of the algebra of smooth functions. (Russian. Russian summary)

Teoret. Mat. Fiz. **140** (2004), *no. 3,* 355–366; *translation in* *Theoret. and Math. Phys.* **140** (2004), *no. 3,* 1195–1204.

The aim of this paper is to compute the Hochschild cohomology of a complex associative algebra of smooth functions on a finite-dimensional real vector space with coefficients in itself.

The author uses a technique, developed by him earlier, which is based on Laplace transformation on the space of cochains. It could be compared with a somewhat similar technique used by I. M. Gel′fand and D. B. Fuchs in their celebrated works on cohomology of infinite-dimensional Lie algebras.

Reviewed by *Pasha Zusmanovich*

MR2117449 (2005i:16013) 16E40 (20C05)

Generalov, A. I. (RS-STPTMM)

Hochschild cohomology of dihedral-type algebras. I. The $D(3K)$ series in characteristic 2. (Russian. Russian summary)

Algebra i Analiz **16** (2004), *no. 6,* 53–122; *translation in St. Petersburg Math. J.* **16** (2005), *no. 6,* 961–1012.

This is continuation of a series of works by the author devoted to Hochschild cohomology of algebras of dihedral type and close to them. These algebras arise in the classification of blocks of group algebras having a tame representation type.

This paper is devoted to a certain series of blocks of a group algebra over an algebraically closed field of characteristic 2, whose defect group is the dihedral group. These algebras admit a description as path algebras of a certain quiver modulo some ideals of relations.

The algebra structure (under the cup product) of the cohomology of an algebra with coefficients in itself is described in terms of generators and relations.

Reviewed by *Pasha Zusmanovich*

MR2093937 (2005g:16071) 16W25 (16S50 47B47)

Nowicki, A. [Nowicki, Andrzej Władysław] (PL-TORNM); **Nowosad, I.** (PL-TORNM)

Local derivations of subrings of matrix rings. (English summary)

Acta Math. Hungar. **105** (2004), *no. 1-2,* 145–150.

For an associative algebra A over a commutative ring, a linear mapping $D \in \mathrm{End}(A)$ is called a local derivation, if for any $a \in A$ there is $d \in \mathrm{Der}(A)$ such that $D(a) = d(a)$.

Finite incidence algebras are algebras of square matrices (of finite size) with zero entries at places whose pair of indices do not belong to a given reflexive transitive relation. In particular, the full matrix algebra and the algebra of upper-triangular matrices are incidence algebras.

In the present paper it is proved that for all finite incidence algebras, each local derivation is a derivation.

Reviewed by *Pasha Zusmanovich*

MR2081925 (2005g:17032) 17B37 (17B50 17B62 17B68)
Grunspan, Cyril (I-ROME-IM)
Quantizations of the Witt algebra and of simple Lie algebras in characteristic p. (English summary)
J. Algebra **280** (2004), *no. 1,* 145–161.

In his landmark lecture notes [*Quantum groups and noncommutative geometry*, Univ. Montréal, Montreal, QC, 1988; MR1016381 (91e:17001)], Yu. I. Manin posed a question about quantization of Witt and Virasoro algebras. Since then, a number of papers appeared tackling this problem, and this is another one.

The problem itself is vague, depending on the precise notion of "quantization". Usually, this is understood as some deformation of structure related to the universal enveloping algebra. The present paper follows this approach—namely, quantization of the Witt algebra is obtained as a deformation of the bialgebra structure on its universal enveloping algebra as defined by E. J. Taft [J. Pure Appl. Algebra **87** (1993), no. 3, 301–312; MR1228159 (94e:17043)] (for alternative approaches, such as deforming the Witt or Virasoro algebra itself, see, e.g., B. A. Khesin, V. V. Lyubashenko and C. Roger [J. Funct. Anal. **143** (1997), no. 1, 55–97; MR1428117 (98f:58104)] and N. H. Hu [Algebra Colloq. **6** (1999), no. 1, 51–70; MR1680657 (2000k:17017)]).

After exploring the (infinite-dimensional) characteristic 0 case, the author provides reduction of his construction modulo p, thus obtaining a deformation of restricted universal enveloping algebra of the modular Witt algebra.

The paper is marred by a number of typos and inaccuracies which, however, could be spotted easily and generally do not reduce its intelligibility and significance.

Reviewed by *Pasha Zusmanovich*

MR2068077 (2005c:17004) 17A40 (17B55)
Kubo, Fujio (J-KYUIT); **Taniguchi, Yoshiaki**
A controlling cohomology of the deformation theory of Lie triple systems. (English summary)
J. Algebra **278** (2004), *no. 1,* 242–250.

The deformation theory of Lie triple systems is presented, following the by nowadays standard format suggested by Gerstenhaber.

The low-dimensional cohomology controlling the deformations is one defined by K. Yamaguti [Kumamoto J. Sci. Ser. A **5** (1960), 44–52 (1960); MR0132770 (24 2606)]. Since the deformed operation is trilinear, the grading is shifted relative to the classical case of deformation of algebras: the infinitesimal deformations are described by the third cohomology, and obstructions to prolongability of infinitesimal deformations lie in the fifth cohomology.

Now, since another deformation theory has been constructed, it would be interesting to enliven it with concrete examples and calculations.

Reviewed by *Pasha Zusmanovich*

MR2062624 (2005c:17026) 17B56 (18G10 18G50)

Inassaridze, N. (GE-AOS); **Khmaladze, E. [Khmaladze, Emzar]** (GE-AOS);
Ladra, M. [Ladra González, Manuel] (E-SACOM-AL)

Non-abelian homology of Lie algebras. (English summary)

Glasg. Math. J. **46** (2004), *no. 2,* 417–429.

In this paper, a nonabelian homology theory of Lie algebras with coefficients in Lie algebras is constructed by means of a derived functor of nonabelian tensor product of Lie algebras.

It generalizes the classical Chevalley-Eilenberg homology, as well as homology with coefficients in crossed module constructed by D. Guin [Ann. Inst. Fourier (Grenoble) **45** (1995), no. 1, 93–118; MR1324126 (96e:18004)].

The authors present properties similar to the usual Chevalley-Eilenberg homology, such as a homology long exact sequence arising from a short exact sequence of Lie algebras. Also, for an associative unital algebra A, a 9-term exact sequence relating the low-dimensional homology of an adjoint Lie algebra $A^{(-)}$ and related algebras with the cyclic homology of A is presented.

An analogous work was previously done for groups [see H. Inassaridze and N. Inassaridze, K-Theory **18** (1999), no. 1, 1–17; MR1710185 (2000g:18019); and references therein].

Reviewed by *Pasha Zusmanovich* (Reykjavík)

MR2157604 (2006f:16019) 16E45 (18G35 18G40 55T15 55U15)

Lapin, S. V.

D_∞**-differentials and** A_∞**-structures in spectral sequences. (Russian)**

Sovrem. Mat. Prilozh. No. 1, Topol., Anal. Smezh. Vopr. (2003), 56–91; *translation in J. Math. Sci. (N. Y.)* **123** (2004), *no. 4,* 4221–4254.

This is an expository article. The author considers a hierarchy of the following situations, generalizing each other in a way too complex to be depicted here.

First, he considers differential modules, their differential perturbations, and the so-called perturbations machine which allows, under certain circumstances, a series of perturbations to be constructed from a given one. This was introduced in works of Gugenheim and later developed by many others in the framework of homological perturbation theory.

Then the author considers generalizations of this situation to D_∞-modules and their perturbations (a notion introduced by the author; this is, roughly, a module with a series of mappings satisfying a generalization of the main equation $d^2 = 0$ of homological algebra), differential (co)algebras and their perturbations (per Gugenheim, Lambe and Stasheff); D_∞-(co)algebras; $(DA)_\infty$-(co)algebras (which generalize the notion of differential A_∞-algebra in the same way that the D_∞-module generalizes the notion of differential module); $(D)_\infty$-modules over $(D)_\infty$-(co)algebras; and, finally, $(DA)_\infty$-modules over $(DA)_\infty$-(co)algebras (per the author).

For the last three notions, it is shown that each suitable generalization of a spectral sequence over a field can be represented as a spectral sequence associated with an appropriate D_∞- or $(DA)_\infty$-structure—the construction of the latter spectral sequence having its origin in a perturbation machine.

Reviewed by *Pasha Zusmanovich*

MR2024088 (2004k:17039) 17B56 (17-04 17B66)

Kornyak, V. V. (RS-JINR-IT)

A method for splitting cochain complexes for computing cohomology: the Lie algebra of Hamiltonian vector fields $H(2|0)$**. (Russian. Russian summary)**

Programmirovanie **2003,** *no. 2,* 48–53; *translation in Program. Comput. Software* **29** (2003), *no. 2,* 94–99.

This is one in a series of papers about the author's highly interesting and ongoing activity in computer calculations of cohomology of Lie algebras and superalgebras with the aid of the program `LieCohomology` developed by him.

Here the author reports about computations of cohomology of the following (ordinary) Lie algebras: the Poisson algebra in two indeterminates and its central extension, and the Lie algebra of Hamiltonian vector fields on the two-dimensional manifold, with coefficients in the trivial module.

The cohomology of these algebras, in the context of computer calculations, attracted early attention [see I. M. Gel′fand, D. I. Kalinin and D. B. Fuks, Funkcional. Anal. i Priložen. **6** (1972), no. 3, 25–29; MR0312531 (47 #1088)], but the structure of the whole cohomology ring still remains a complete mystery.

The algorithm developed by the author consists in singling out recursively a subcomplex in the initial cochain complex and a corresponding reduction of the remaining complex, so the whole problem splits into calculations of cohomology of smaller subcomplexes.

The result is a calculation of cohomology up to degree 12 with gradings between -2 and 8 (with respect to cohomology grading induced by the standard grading of Lie algebras under consideration), which is a slight improvement of the 1972 Gel′fand-Kalinin-Fuks result.

At the end the author outlines possible improvements, some of which (like performing calculations in modular arithmetic instead of in rational numbers) are already being implemented by him in subsequent publications (see ["On the structure of cohomology of Hamiltonian p-algebras", preprint, arXiv.org/abs/math/0404247] and other preprints of the author on arXiv).

It should be noted that these calculations seem to be highly parallelizable, and, in the reviewer's opinion, this could be one of the major avenues for further improvements.

Also, the impact and usability of this and other papers of the author would be increased drastically if they, like numerous other software projects, were accompanied by a website offering the program source code, full results of calculations, and all other voluminous but highly relevant details which cannot be reproduced in the journal article.

In the absence of such, one cannot say much on the programming aspects of this work, except that the program is written in C and runs on Wintel machines.

Reviewed by *Pasha Zusmanovich*

MR1998044 (2004f:17003) 17A36

Jiménez-Gestal, Clara (E-LARI-CP); **Pérez-Izquierdo, José M.** (E-LARI-CP)

Ternary derivations of generalized Cayley-Dickson algebras. (English summary)

Comm. Algebra **31** (2003), *no. 10,* 5071–5094.

A ternary derivation of a (nonassociative) algebra A is a triple (d_1, d_2, d_3) of endomorphisms of A such that $d_1(xy) = d_2(x)y + xd_3(y)$. Analogously one defines a ternary automorphism. One can argue that these are just further pointless generalizations of the standard notions of derivation and automorphism, but the authors provide convincing arguments showing that these notions are in a sense natural and worth studying.

R. D. Schafer [Amer. J. Math. **76** (1954), 435–446; MR0061098 (15,774d)] described derivations of the Cayley-Dickson algebras (that is, the infinite series of algebras with involution of dimension 2^n, obtained via recursive doubling process, whose initial terms are celebrated quaternions and octonions). Particularly, he proved that the derivation algebra stabilizes at the third term in a series.

Here the authors generalize this half-century-old result by describing ternary derivations of the Cayley-Dickson algebras (over an arbitrary field of characteristic $\neq 2, 3$). In contrast with Schafer's results, they discover that ternary derivations stabilize at the fourth term.

They also compute the ternary automorphisms of the Cayley-Dickson algebras in terms of (ordinary) automorphisms. Automorphisms of the Cayley-Dickson algebras were earlier described by P. M. Eakin, Jr. and A. Sathaye [J. Algebra **129** (1990), no. 2, 263–278; MR1040939 (91b:17002)]. In contrast with derivations, ternary automorphisms (as well as ordinary ones) never stabilize.

Reviewed by *Pasha Zusmanovich*

MR1970052 (2004c:17007) 17B05 (17B30)

Barnes, Donald W.; **Groves, Daniel** (5-ANU-SAS)

The Wielandt subalgebra of a Lie algebra. (English summary)

J. Aust. Math. Soc. **74** (2003), *no. 3,* 313–330.

For a Lie algebra L, the authors define a Wielandt subalgebra $\omega(L)$ as the intersection of the normalizers of all subnormal subalgebras of L. This is a straightforward analogue of a well-known notion from group theory.

If the characteristic of the ground field is zero, or the derived algebra L' is nilpotent, then $\omega(L)$ is an ideal of L, which in these cases allows one to define a Wielandt series recursively by $\omega_1(L) = \omega(L)$, $\omega_{i+1}(L)/\omega_i(L) = \omega(L/\omega_i(L))$.

The paper under review contains numerous results about Wielandt subalgebras and Wielandt series. In particular, bounds for a Wielandt length (the length of the Wielandt series) are obtained and Lie algebras of Wielandt length 2 are described.

There are also examples showing how the matters become (as usual) more complicated in characteristic p.

Reviewed by *Pasha Zusmanovich*

MR1937401 (2003j:17024) 17B50 (17A99 17B35)
Koreshkov, N. A. (RS-KAZA)
Casimir elements of $\mathbb{Z}$-forms of modular Lie algebras. (Russian)
Izv. Vyssh. Uchebn. Zaved. Mat. **2002,** *no. 3,* 32–35; *translation in Russian Math. (Iz. VUZ)* **46** (2002), *no. 3,* 28–31.

All algebras are assumed to be finite-dimensional, and defined over an arbitrary field. Let G be a (not necessarily Lie or associative) algebra, and $S(G)$ be the symmetric algebra over G. Suppose that $S(G)$ contains, as a G-module under left multiplications, a submodule P whose dual P^* is also embedded in $S(G)$. Let $\{u_i\}$, $\{u_i^*\}$ be the dual bases of P and P^* respectively. Then $\sum_i u_i u_i^* \in S(G)^G$ is called a generalized Casimir element.

In the present paper, the author proves that any homogeneous element of $S(G)^G$ is a generalized Casimir element (under an additional restriction on the degree of an element in the case of positive characteristic).

Note that it has been conjectured [A. S. Dzhumadil′daev, Izv. Akad. Nauk SSSR Ser. Mat. **49** (1985), no. 5, 1107–1117, 1120; MR0810531 (87g:17014)] that any element of the center $Z(U(L))$ of a universal enveloping algebra of a modular Lie algebra L is a generalized Casimir element (defined as above by replacing the symmetric algebra by a universal enveloping algebra).

Though the author claims that his result proves Dzhumadil′daev's conjecture, this is not exactly the case, as the relationship between $S(L)^L$ and $Z(U(L))$, known as the "Duflo isomorphism" in the zero characteristic case, is not so apparent in the case of positive characteristic, as, for example, was noted by the author himself in a recent related publication [Izv. Vyssh. Uchebn. Zaved. Mat. **2002**, no. 7, 22–26; MR1943603 (2003j:17025)].

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Reviewed by *Pasha Zusmanovich*

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MR1943603 (2003j:17025) 17B50 (17B35)
Koreshkov, N. A. (RS-KAZA)
Central elements and invariants in modular Lie algebras. (Russian)
Izv. Vyssh. Uchebn. Zaved. Mat. **2002,** *no. 7,* 22–26; *translation in* *Russian Math. (Iz. VUZ)* **46** (2002), *no. 7,* 20–24.

For a finite-dimensional Lie algebra L over a field of zero characteristic, there is a well-known L-module isomorphism between the center of a universal enveloping algebra $Z(U(L))$ and invariants of the symmetric algebra over L under the L-action $S(L)^L$ [M. Duflo, Ann. Sci. École Norm. Sup. (4) **10** (1977), no. 2, 265–288; MR0444841 (56 #3188)].

In this paper, an analogue for the positive characteristic case is proved. The result is not as generic as in the zero characteristic case and technically more complicated, as it involves passing to an extension over the base field, a ring of quotients of the ring of symmetric invariants and some subring of the center of a universal enveloping algebra.

Reviewed by *Pasha Zusmanovich*

MR1936584 (2003j:17001) 17A30 (17A32)
Majumdar, Anita (6-ISI-SM); **Mukherjee, Goutam** (6-ISI-SM)
Deformation theory of dialgebras. (English summary)
K-Theory **27** (2002), *no. 1,* 33–60.

Dialgebras were introduced by J.-L. Loday [C. R. Acad. Sci. Paris Sér. I Math. **321** (1995), no. 2, 141–146; MR1345436 (96f:16013)]. They are vector spaces endowed with two binary linear operations satisfying 5 associativity-like axioms. Roughly speaking, dialgebras play the same role to Leibniz algebras as associative algebras to Lie algebras.

Cohomology theory (with nontrivial coefficients) of dialgebras was developed by A. Frabetti [in *Dialgebras and related operads*, 67–103, Lecture Notes in Math., 1763, Springer, Berlin, 2001; MR1860995 (2002i:17003)].

In the present paper, a deformation theory of dialgebras is constructed. It follows a nowadays standard format suggested by Gerstenhaber: infinitesimal deformations are controlled by the second cohomology group with coefficients in an adjoint module, and obstructions to their prolongability are controlled by the third one.

Maybe the biggest difference with the classical deformation theory of associative algebras is that since dialgebras have 2 operations satisfying 5 axioms, all computational efforts are at least 5 times more intensive.

{For additional information pertaining to this item see [A. Majumdar and G. Mukherjee, K-

Theory **35** (2005), no. 3-4, 395–397 (2006); MR2240239].}

REVISED (October, 2006)

Current version of review. Go to earlier version.

Reviewed by *Pasha Zusmanovich*

MR1918187 (2003f:17019) 17B50 (17B20 17B56)

Dzhumadil′daev, A. S.; **Ibraev, S. S.** (KZ-AOS)

Nonsplit extensions of modular Lie algebras of rank 2. (English summary)

The Roos Festschrift volume, 1.

Homology Homotopy Appl. **4** (2002), *no. 2, part 1,* 141–163.

As was noted on numerous occasions, the cohomological behaviour of modular Lie algebras is very different from the zero characteristic case. In particular, it was shown that for any finite-dimensional modular Lie algebra the number of cohomologically nontrivial irreducible modules is finite and nonzero. Thus the problem of the description of such modules for any given algebra is "tame" in a sense.

The present paper, along with the previous research on this theme, shows that, although being "tame", it is quite difficult.

Here the authors exhibit all irreducible modules with non-vanishing second cohomology for classical Lie algebras A_2, B_2 and G_2 and under the additional restriction that the characteristic of the ground field is greater than the corresponding Coxeter number.

This continues the previous research of Dzhumadildaev and his collaborators, where such modules were described for $\mathrm{sl}(2)$ and some initial algebras in Cartan series.

The proof is quite technical and complex and uses such devices as restricted cohomology and the first Frobenius kernel.

{Reviewer's remark: For a recent analogous work for algebraic groups, see [G. J. McNinch, Pacific J. Math. **204** (2002), no. 2, 459–472 MR1907901 (2003m:20062)].}

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR1883481 (2002k:17038) 17B56
Khmaladze, Emzar (GE-AOS)
Homology of Lie algebras with $\Lambda/q\Lambda$ coefficients and exact sequences. (English summary)
Theory Appl. Categ. **10** (2002), 113–126.

Let L be a Lie algebra over a commutative ring Λ, I an ideal of L. G. J. Ellis [J. Pure Appl. Algebra **46** (1987), no. 2-3, 111–115; MR0897010 (88k:17016)] has obtained the following 6-term exact sequence: $\mathrm{Ker}(L \wedge I \to I) \to H_2(L) \to H_2(L/I) \to I/[L, I] \to H_1(L) \to H_1(L/I) \to 0$, where $H_i(-)$ is a Lie algebra homology with coefficients in Λ and $\wedge$ is a nonabelian exterior product of Lie algebras. (When the ground ring Λ is a field, the 5 right terms constitute the well-known 5-term exact sequence arising from the Hochschild-Serre spectral sequence.)

In this paper, the author generalizes this 6-term exact sequence in two ways. First, he extends it to the left by another two terms

$$H_3(L) \to H_3(L/I) \to \dots$$

and second, he proves the result for homology with coefficients in $\Lambda/q\Lambda$, where q is a nonnegative integer.

As a consequence, he derives the previously known Hopf formulas expressing the second and third homology of a Lie algebra in terms of its presentation, for the case of homology with coefficients in $\Lambda/q\Lambda$.

The main technique used is a nonabelian derived functor of exterior products of Lie algebras modulo q, and its relationship with the Lie algebra homology.

Reviewed by *Pasha Zusmanovich*

MR1898599 (2003b:17010) 17B30
Echarte, F. J. [Echarte Reula, Francisco Javier] (E-SEVLM-GM);
Núñez, J. [Núñez-Valdés, Juan] (E-SEVLM-GM); **Ordóñez, M.** (E-SEVLM-GM)
Derived filiform Lie algebras having first coefficient 0. (English summary)
An. Univ. Bucureşti Mat. **49** (2000), *no. 2,* 141–154.

A finite-dimensional nilpotent Lie algebra L is called filiform, if it possesses a longest possible lower central series L^k (that is, $\dim\ L^k = \dim\ L - k,\ 2 \le k \le \dim\ L$). This notion was introduced by M. Vergne [Bull. Soc. Math. France **98** (1970), 81–116; MR0289609 (44 #6797)] who proved that, in a sense, "most" nilpotent Lie algebras are filiform.

In this paper, the authors continue their studies of complex filiform Lie algebras. They investigate when a filiform Lie algebra, subject to some additional technical condition, is a commutant of a (solvable) Lie algebra, and classify such filiform algebras.

Reviewed by *Pasha Zusmanovich*

MR1272372 (95b:17027) 17B56 (17B66)

Lecomte, P. B. A. (B-LIEG-IM); **Roger, C. [Roger, Claude]** (F-LYON)

Remarques sur la cohomologie de l'algèbre de Nijenhuis-Richardson. (French) [Remarks on the cohomology of the Nijenhuis-Richardson algebra]

Comm. Algebra **22** (1994), *no. 8,* 3053–3059.

For a finite-dimensional vector space E over a field of characteristic zero, the authors consider a $\mathbf{Z}$-graded Lie algebra (sometimes called a color Lie superalgebra) $A(E)$ defined by A. Nijenhuis and R. Richardson in the 1960s in their works on Lie algebra deformations. $A(E)$ can also be recognized as a Lie superalgebra $W(0,n)$ of tangent vector fields to a $(0,n)$-dimensional supervariety, as well as a (graded) derivation algebra of a commutative graded algebra $\bigwedge E^*$ (as a vector space, $A(E) \simeq \bigwedge E^* \otimes E$).

They compute the cohomology of $A(E)$ with coefficients in the modules $\Omega^p(E) = \bigwedge E^* \otimes S^p E^*$ and $\Omega_p(E) = \bigwedge E^* \otimes S^p E$ (particularly, $\Omega_1(E)$ is the adjoint module).

The methods of computation are standard pleasant methods of homological algebra—the Hochschild-Serre spectral sequence and Shapiro's lemma (called by the authors the coinduction lemma) relating the cohomology with coefficients in a coinduced module with the cohomology of a subalgebra.

One of the consequences of these results is rigidity (absence of deformations) of $A(E)$.

Reviewed by *Pasha Zusmanovich*

MR1255711 (95b:17025) 17B55 (16E40 16S80 16W25 17B35)
Coll, Vincent; **Gerstenhaber, Murray** (1-PA); **Schack, Samuel D.** (1-SUNYB)
Universal deformation formulas and breaking symmetry. (English summary)
J. Pure Appl. Algebra **90** (1993), *no. 3,* 201–219.

Universal deformation formulas (UDFs) are, roughly speaking, ways to deform an (associative unital) algebra A in the presence of an action of a Lie algebra $\mathfrak{g}$ as derivations of A [resp. a Lie group G as automorphisms of A], depending only on $\mathfrak{g}$ [resp. on G] and not on A. Strictly speaking, UDFs based on a Lie algebra $\mathfrak{g}$ are certain elements of the tensor algebra $T^\bullet(U\mathfrak{g})$ of the universal enveloping algebra $U\mathfrak{g}$ such that if $\mathfrak{g}\to\operatorname{Der}(A,A)$ is a Lie algebra morphism then the induced map $T^\bullet(U\mathfrak{g})\to C^\bullet(A,A)$ carries them to a deformation of A.

The authors develop a theory of UDFs parallel to the classical theory of deformations of Lie algebras. In particular, there is a notion of equivalent UDFs such that the second cohomology group $H^2(T^\bullet(U\mathfrak{g}))\simeq\bigwedge^2(\mathfrak{g})$ (the latter isomorphism itself is one of the basic results of the paper) is responsible for "infinitesimals" of UDFs, and obstructions to "extending" UDFs lie in the image of the "square map" $\operatorname{Sq}\colon\bigwedge^2(\mathfrak{g})\to\bigwedge^3(\mathfrak{g})$. The authors exhibit concrete examples of UDFs for two-dimensional abelian and nonabelian Lie algebras, for $\mathfrak{sl}(n)$ and for the Heisenberg algebra.

There are also results about a maximal number of "prolongation steps" for "infinitesimals" (elements from $\bigwedge^2\mathfrak{g}$) for a nilpotent Lie algebra $\mathfrak{g}$, to guarantee its "full integrability" to a UDF in terms of an index of nilpotency of $\mathfrak{g}$ and a pleasant conjecture that this number is 1 for a classical semisimple Lie algebra $\mathfrak{g}$ (which is verified for $\mathfrak{sl}(2)$).

In the final part of the paper the authors consider the phenomenon when an automorphism or derivation of an algebra does not prolong to one of a deformed algebra, i.e. "breaks" (unlike in previous considerations, the ground field is assumed to be algebraically closed of characteristic zero). The driving conjecture here is the following: If a non-nilpotent Lie algebra $\mathfrak{g}$ acts by outer derivations on an algebra A, then every non-nilpotent element of $\mathfrak{g}$ breaks under some deformation of A.

What is actually proved is that if $\mathfrak{g}$ contains a two-dimensional nonabelian Lie algebra (which is equivalent to being non-nilpotent), then there is a UDF based on $\mathfrak{g}$ which breaks some automorphism/derivation of A. To quote the authors, "it is curious that the very existence of a non-nilpotent symmetry group generally provides a path to the breaking of symmetry!"

Though the theory presented is heavily based on the previously developed deformation theory of algebras, all necessary definitions, results, historical remarks, etc., are given, so the paper can be read independently. It contains a fascinating newly developed piece of deformation theory, poses many questions and may and should serve as a source of further developments.

Reviewed by *Pasha Zusmanovich*

MR1254727 (94k:17038) 17B50 (17B10)

Kozerenko, K. V. (RS-SL)

Main theorems of invariant theory for the Lie algebra $\mathfrak{sl}(2)$ in the case of a field of finite characteristic.

Unconventional Lie algebras, 75–102, *Adv. Soviet Math.*, 17, *Amer. Math. Soc., Providence, RI*, 1993.

Let V be a two-dimensional module over a modular Lie algebra $\mathrm{sl}(2)$ and V^* be its adjoint module. The author describes fully the space of invariants of the $\mathrm{sl}(2)$-module $\underbrace{V\otimes\cdots\otimes V}_{k\text{ times}}\otimes\underbrace{V^*\otimes\cdots\otimes V^*}_{l\text{ times}}$. Sophisticated combinatorics is involved in the proof.

{For the entire collection see MR1254723 (94f:17001)}

Reviewed by *Pasha Zusmanovich*

MR1242837 (94i:17009) 17B01 (17B40)

Papistas, A. I. (GR-IOAN)

Automorphisms of free polynilpotent Lie algebras.

Comm. Algebra **21** (1993), *no. 12,* 4391–4395.

The author proves that some classes of relatively free Lie algebras have nontame automorphisms (that is, automorphisms which are not induced by automorphisms of the corresponding absolutely free Lie algebra). Similar results were obtained earlier for groups by G. K. Gupta and F. Levin [Comm. Algebra **19** (1991), no. 9, 2497–2500; MR1125185 (92k:20055)] and V. È. Shpil′raĭn [Internat. J. Algebra Comput. **1** (1991), no. 2, 177–184; MR1128010 (92k:20056)]

The author also states without proof a generalization of the Gupta-Levin result.

Reviewed by *Pasha Zusmanovich*

MR1241179 (94i:17028) 17B50 (17B67)

Skryabin, S. M.

A contragredient 29-dimensional Lie algebra of characteristic 3.
(Russian. English, Russian summaries)

Sibirsk. Mat. Zh. **34** (1993), *no. 3,* 171–178, 223, 228; *translation in Siberian Math. J.* **34** (1993), *no. 3,* 548–554.

The author shows that the 29-dimensional simple Lie algebra of characteristic 3 constructed earlier by G. Brown [Math. Ann. **261** (1982), no. 4, 487–492; MR0682662 (84f:17008)] is a contragredient Lie algebra. This implies that the list of simple contragredient Lie algebras of characteristic 3, provided by B. Weisfeiler and V. G. Kac [Izv. Akad. Nauk SSSR Ser. Mat. **35** (1971), 762–788; MR0306282 (46 #5408)], is not complete.

Reviewed by *Pasha Zusmanovich*

MR1220061 (94d:17028) 17B56 (17B10 17B50)

Chiu, Sen [Qiu, Sen] (PRC-ECNU)

The cohomology of modular Lie algebras with coefficients in a restricted Verma module.
(English summary)

A Chinese summary appears in Chinese Ann. Math. Ser. A **14** (1993), no. 1, 135.

Chinese Ann. Math. Ser. B **14** (1993), *no. 1,* 77–84.

Let $\mathfrak{g}$ be a classical modular semisimple Lie algebra, $\mathfrak{b}$ a Borel subalgebra, $Z(\lambda)$ a restricted (finite-dimensional) Verma module over $\mathfrak{g}$ with a highest weight λ. The goal of this paper is to compute $H^*(\mathfrak{g}, Z(\lambda))$. The proof goes along the lines of the zero characteristic case [F. L. Williams, Trans. Amer. Math. Soc. **240** (1978), 115–127; MR0486012 (58 #5804)] and generalizes an earlier result of R. Farnsteiner and H. Strade [Math. Z. **206** (1991), no. 1, 153–168; MR1086821 (92d:17018)] about sufficient conditions for the vanishing of $H^*(\mathfrak{g}, Z(\lambda))$. The main ingredient of the proof is a modular version of Shapiro's lemma, proved independently by A. S. Dzhumadil′daev [Mat. Sb. **180** (1989), no. 4, 456–468, 559; MR0997895 (90e:17026)] and by Farnsteiner and Strade [op. cit.], which allows the author to reduce the problem to the computation of cohomology of the nilradical of $\mathfrak{b}$.

Reviewed by *Pasha Zusmanovich*

MR1210183 (94j:17009) 17B35 (16W30 17B50)

Qiu, Sen (PRC-ECNU)

On central elements of the restricted universal enveloping algebra of a restricted Cartan type Lie algebra. (English summary)

Northeast. Math. J. **8** (1992), *no. 3,* 311–316.

Let L be a modular restricted simple Lie algebra of Cartan type W_n, S_n or H_n, L_0 a zero term in the standard **Z**-grading of L (known to be isomorphic to $\mathrm{gl}(n)$, $\mathrm{sl}(n)$ or $\mathrm{sp}(n)$ respectively), $u(L)$ and $u(L_0)$ restricted universal enveloping algebras of L and L_0 (considered as Hopf algebras). Using A. S. Dzhumadil′daev's construction of central elements in $u(L)$ [Izv. Akad. Nauk SSSR Ser. Mat. **49** (1985), no. 5, 1107–1117, 1120; MR0810531 (87g:17014)] (so-called Casimir elements) and W. J. Haboush's construction of the integral in $u(L_0)$ [in *Séminaire d'Algèbre Paul Dubreil et Marie-Paule Malliavin, 32ème année (Paris, 1979),* 35–85, Lecture Notes in Math., 795, Springer, Berlin, 1980; MR0582073 (82a:20049)], the author computes the integral in $u(L)$ (which is unique up to multiplication by scalars). This provides an example of a central element in $u(L)$ different from those found by Dzhumadil′daev. Recall in this connection Dzhumadil′daev's conjecture that $u(L)$ is generated (as an ordinary algebra) by Casimir elements.

Reviewed by *Pasha Zusmanovich*

MR1209955 (94e:17038) 17B66 (17-08 17B56)

Kochetkov, Yu. Yu. (RS-INT); **Post, G. F.** (NL-TWEN)

Deformations of the infinite-dimensional nilpotent Lie algebra L_2. (Russian)

Funktsional. Anal. i Prilozhen. **26** (1992), *no. 4,* 90–92; *translation in Funct. Anal. Appl.* **26** (1992), *no. 4,* 304–305 (1993).

Let W be an infinite-dimensional Lie algebra over **C** with a basis $\{e_i\}$, $i = -1, 0, 1, \cdots$, and with multiplication table $[e_i, e_j] = (j - i)e_{i+j}$. This is the famous Witt algebra. It possesses a filtration

$W = L_{-1} \supset L_0 \supset L_1 \supset \cdots$, where L_k is defined as a linear span of $\{e_i\}$, $i \geqslant k$. It is known that the algebras W and L_0 are rigid [D. B. Fuks, *Cohomology of infinite-dimensional Lie algebras*, English translation, Consultants Bureau, New York, 1986; MR0874337 (88b:17001)]. Deformations of the Lie algebra L_1 were described by A. Fialowski [in *Deformation theory of algebras and structures and applications (Il Ciocco, 1986)*, 375–401, Kluwer Acad. Publ., Dordrecht, 1988; MR0981622 (90c:17027)]. She also proved [Studia Sci. Math. Hungar. **27** (1992), no. 1-2, 189–200; MR1207571 (93k:17043)] that the dimension of $H^2(L_2, L_2)$ (the space of infinitesimal deformations of L_2) lies between 8 and 23.

The aim of the paper under review is to describe deformations of L_2. The grading $L_2 = \bigoplus_{i=2}^{\infty} \mathbf{C}e_i$ is inherited by the cohomology group $H^*(L_2, L_2)$. The authors discover that $H^2(L_2, L_2)$ is 8-dimensional and exhibit the basic cocycles in each graded component. It turns out that each infinitesimal deformation defined by such a cocycle is either unextendable to a global one, or its Massey square is zero (i.e. it is extendable "trivially").

The computations are sketched very briefly.

Reviewed by *Pasha Zusmanovich*

MR1209159 (94e:17035) 17B56 (17B10 17B20)
Manturov, O. V. (RS-MPDI)

Cohomology of a class of Lie algebras. (Russian)

Uspekhi Mat. Nauk **47** (1992), *no. 6(288),* 221–222; *translation in Russian Math. Surveys* **47** (1992), *no. 6,* 224–225.

The author deals with the question of the computation of cohomology with trivial coefficients of a finite-dimensional Lie algebra over a field of characteristic zero in terms of its Levi decomposition. Since the cohomology of semisimple Lie algebras is well known, the next natural step is to consider Lie algebras whose radical is not too complicated. The cohomology of the Lie algebra $L = S \oplus R$, where S is a semisimple Lie subalgebra acting irreducibly on a radical R, was considered earlier by the author [Trudy Sem. Vektor. Tenzor. Anal. No. 24 (1991); per bibl.]. Here he considers Lie algebras whose radical decomposes as an S-module into the sum of two irreducible components R_1 and R_2. It is shown that the problem of the computation of cohomology of such algebras is reduced to the problem of finding S-invariants of the tensor product of modules of skew-symmetric polylinear forms on R_1 and R_2, and two concrete examples are provided (with $S = \mathrm{sp}(n)$ and $\mathrm{sl}(2)$).

Reviewed by *Pasha Zusmanovich*

MR1179371 (94a:17021) 17B68 (81R10)
Zha, Chao Zheng (PRC-XINJ-P)
High order differential neighbourhoods in holomorphic mappings. (English summary)
Phys. Lett. B **288** (1992), *no. 3-4,* 269–272.

The author shows that the kth order differential neighborhoods of the holomorphic mappings from S^1 to a vector space are generated by the kth order Virasoro operators $L^k_m = (-1)^k z^{m+k} d^k/dz^k$. He provides an explicit multiplication formula for the Lie algebra generated by these operators, and determines its central extensions, getting therefore a generalization of the Virasoro algebra.

{Reviewer's remarks: There is a large volume of literature devoted to various generalizations of the Virasoro algebra [see, e.g., W. L. Li, J. Algebra **122** (1989), no. 1, 64–80; MR0994935 (90d:17018); M. Schlichenmaier, "Verallgemeinerte Krichever-Novikov Algebren und deren Darstellungen", Dissertation, Univ. Mannheim, Mann- heim, 1990; Zbl 721:30031; J. Patera and H. Zassenhaus, Comm. Math. Phys. **136** (1991), no. 1, 1–14; MR1092566 (92e:17038); A. O. Radul, Funktsional. Anal. i Prilozhen. **25** (1991), no. 1, 33–49; MR 92k: 17039].}

Reviewed by *Pasha Zusmanovich*

MR1163017 (93f:17033) 17B56 (17B10)
Bavula, V. V. (UKR-KIEV)
Calculation of $H^*(\mathrm{sl}(2), M)$ with coefficients in a simple $\mathrm{sl}(2)$-module. (Russian)
Funktsional. Anal. i Prilozhen. **26** (1992), *no. 1,* 57–58; *translation in* *Funct. Anal. Appl.* **26** (1992), *no. 1,* 45–46.

The ground field is assumed to be algebraically closed of characteristic zero. The classification of simple (not nessessarily finite-dimensional) $\mathrm{sl}(2)$-modules was done by R. E. Block [Bull. Amer. Math. Soc. (N.S.) **1** (1979), no. 1, 247–250; MR0513751 (80i:17008)] and later refined by the author [Ukrain. Mat. Zh. **42** (1990), no. 9, 1174–1180; MR1093625 (92c:17009)]. In this paper, the formulas for cohomology of $\mathrm{sl}(2)$ with coefficients in simple modules are presented. Since $H^0(\mathrm{sl}(2), M) = 0$ for a nontrivial simple module M, the Poincaré duality implies $H^3(\mathrm{sl}(2), M) = 0$, so actually the author computes H^1 and H^2.

Among simple weight modules, there are only 3 cohomologically nontrivial modules: the trivial module and two Verma modules with highest [resp. lowest] weight equal to -2 [resp. $+2$]. For these modules explicit values of basic cocycles are given. For a simple nonweight module, the formulas for cohomology in terms of action of certain elements of the universal enveloping algebra $U(\mathrm{sl}(2))$ are presented. As a corollary of these formulas, the cohomological triviality of Whittaker modules, introduced elsewhere [D. Arnal and G. Pinczon, J. Math. Phys. **15** (1974), 350–359; MR0357527 (50 #9995)], is derived.

The proofs are either absent, or sketched very briefly.

Reviewed by *Pasha Zusmanovich*

MR1162129 (93f:17035) 17B56 (17B66 17B70)

Chiu, Sen [Qiu, Sen] (PRC-ECNU)

Cohomology of graded Lie algebras of Cartan type of characteristic 0.

A Chinese summary appears in Acta Math. Sinica **36** (1993), no. 2, 288.
Acta Math. Sinica (N.S.) **8** (1992), *no. 1,* 17–25.

The ground field is assumed to be algebraically closed of characterictic zero. This paper is devoted to the computation of the first cohomology group of the simple, graded of depth 1, infinite-dimensional Lie algebras of Cartan type (as known, such algebras are divided into three series: general, special and Hamiltonian) with coefficients in certain graded modules.

The author shows that the so-called mixed product modules, introduced elsewhere [G. Y. Shen, Sci. Sinica Ser. A **29** (1986), no. 6, 570–581; MR0862418 (88d:17014)], are isomorphic to the graded coinduced modules from the finite-dimensional modules over the nonnegative component in the standard grading (Theorem 1.2).

Combining the standard long exact sequences for the low-dimensional Lie algebra cohomology with the known facts about the cohomology of coinduced modules, the author gets an expression of the first cohomology group with the coefficients in mixed products (Proposition 2.5), and using their relationship with graded irreducible modules, determines, via case-by-case study, the first cohomology group with coefficients in the latter ones.

{Reviewer's remarks: The reference to a book of D. B. Fuks (the name misspelled in the paper) [*Cohomology of infinite-dimensional Lie algebras*, English translation, Consultants Bureau, New York, 1986; MR0874337 (88b:17001)] is a little bit misleading: actually this book is devoted to the celebrated works of I. M. Gel′fand and his collaborators and pupils on the structure of the whole cohomology ring of the Lie algebras of vector fields. For related results in the cohomology of modular Lie algebras of Cartan type (not necessarily of the first order) see papers by A. S. Dzhumadild′aev [Mat. Sb. (N.S.) **119(161)** (1982), no. 1, 132–149; MR0672414 (83m:17008)] and R. Farnsteiner [Math. Ann. **288** (1990), no. 4, 713–730; MR1081272 (92b:17027)].}

Reviewed by *Pasha Zusmanovich*

MR1179213 (93m:17007) 17B35 (17B50)

Koreshkov, N. A.

An invariant of the algebra W_n. (Russian)

Izv. Vyssh. Uchebn. Zaved. Mat. **1991,** *no. 10,* 40–42; *translation in Soviet Math. (Iz. VUZ)* **35** (1991), *no. 10,* 36–38.

This short paper is devoted to the question of finding central elements in the universal enveloping algebra of the modular Lie algebra of Cartan type W_n. The algebra W_n is defined as a derivation algebra of the ring $K[x_1,\cdots,x_n]/(x_1^p-1,\cdots,x_n^p-1)$, where p is a characteristic of the ground field K (when K is perfect, this coincides with the usual definition when the polynomial ring is factorized through the ideal $(x_1^p,\cdots,x_n^p)$).

The principal tools are formulas describing the W_n-action on the tensor algebra $T(W_n)$. In the case $n=1$ these formulas allow one to obtain a certain invariant of this action, whose image under factorization is a central element of $U(W_1)$. However, this does not lead to a new result (at least in the case of the perfect ground field), since the center of W_1 was determined by N. N. Yakovlev [Funktsional. Anal. i Prilozhen. **6** (1972), no. 2, 99–100; MR0301067 (46 #225)].

In the general case the author constructs invariants of the "restricted symmetric algebra" $\overline{S}(L)=S(L)/(x^{[p]}-x)$, $L=W_n$. He remarks that under the assumption that W_n-modules $S(W_n)$ and $U(W_n)$ are isomorphic, this gives rise to the central elements of the restricted universal enveloping algebra $\overline{U}(W_n)$. (This assumption was claimed to be true for any finite-dimensional restricted Lie algebra by A. A. Mil′ner [Funktsional. Anal. i Prilozhen. **14** (1980), no. 2, 67–68; MR0575217 (81h:17012)], but its failure in the general case was shown by E. M. Friedlander and B. J. Parshall [Ann. Sci. École Norm. Sup. (4) **20** (1987), no. 2, 215–226; MR0911755 (88k:14026)].)

Reviewed by *Pasha Zusmanovich*