MR2673758 (2011h:17038) 17B69 (18D50)

Hortsch, Ruthi (1-MI); **Kriz, Igor** (1-MI); **Pultr, Aleš** (CZ-KARL-AM)

A universal approach to vertex algebras. (English summary)

J. Algebra **324** (2010), *no. 7,* 1731–1753.

There are two widespread and somewhat contradictory opinions: first, vertex algebras are one of the fundamental structures in modern mathematics; second, fundamental mathematical structures cannot have technical-looking definitions comprising more than one page. In fact, the notion of vertex algebra, since its introduction by Borcherds, has constantly evolved in an attempt to reconcile these two opinions. The present paper is another step in this direction: the authors characterize vertex algebras as algebras over a certain graded co-operad.

Reviewed by *Pasha Zusmanovich*

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Article

MR2668096 (2011h:17030) 17B62 (17B60 17C50 17D05)

Goncharov, M. E. (RS-NOVO-MM)

Lie bialgebras that arise from alternative and Jordan bialgebras.
(Russian. Russian summary)

Sibirsk. Mat. Zh. **51** (2010), *no. 2,* 268–284; *translation in* *Sib. Math. J.* **51** (2010), *no. 2,* 215–228.

By analogy with Lie theory, the author considers the notion of bialgebra, Drinfeld double and Yang-Baxter equation for an arbitrary variety of (nonassociative) algebras, establishes relationships between some classes of alternative, Jordan and Lie bialgebras, similar to the classical Kantor-Koecher-Tits construction, and proves that every finite-dimensional alternative algebra over an algebraically closed field of characteristic $\neq 2$ admits a nontrivial structure of a quasitriangular alternative bialgebra.

Reviewed by *Pasha Zusmanovich*

MR2666994 (2011e:17016) 17B25 (17B20 17B40 17B70 17D05)
Calderón Martín, A. J. (E-CADS);
Draper, C. [Draper Fontanals, Cristina] (E-MALIE-AM);
Martín González, C. (E-MALS-GT)
Gradings on the real forms of the Albert algebra, of $\mathfrak{g}_2$, and of $\mathfrak{f}_4$. (English summary)
J. Math. Phys. **51** (2010), *no. 5,* 053516, 21 *pp*.

A grading of an algebra by an abelian group is called fine if its homogeneous components cannot be further decomposed to get another, "finer" grading.

The purpose of this article is to describe fine gradings, together with their realizations, of real forms of simple exceptional Lie algebras of type G_2 and F_4, and of the alternative Albert algebra.

The authors demonstrate two approaches to this problem: one, more concrete, through conjugacy classes of certain abelian subgroups of automorphism groups of the corresponding algebras, and another, more generic, through affine algebraic group schemes and comodule maps.

The article also contains clearly written and useful generalities about the relationship between gradings of complex simple Lie algebras and their real forms.

Reviewed by *Pasha Zusmanovich*

MR2662406 (2011e:16012) 16E40 (16S34)
Generalov, A. I. (RS-STPT)
Hochschild cohomology for algebras of dihedral type. III. Local algebras in characteristic 2. (English summary)
Vestnik St. Petersburg Univ. Math. **43** (2010), *no. 1,* 23–32.

Using a bimodule resolution for a series of local algebras of dihedral type constructed by the author in the first paper in the series [Algebra i Analiz **16** (2004), no. 6, 53–122; MR2117449 (2005i:16013)], the Hochschild cohomology with coefficients in itself is computed for a certain family of such algebras in characteristic 2 (which includes the group algebra of the dihedral group of order 2^n).

{For Part II see [A. I. Generalov, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov.

(POMI) **375** (2010), 92–129].}

Reviewed by *Pasha Zusmanovich*

MR2650373 (2011d:17050) 17B69 (17C20 17C50)
Niibori, Hidekazu (J-TSUKS-GAS); **Sagaki, Daisuke** (J-TSUKS-IM)
Simplicity of a vertex operator algebra whose Griess algebra is the Jordan algebra of symmetric matrices. (English summary)
Comm. Algebra **38** (2010), *no. 3,* 848–875.

It is known that to every vertex operator algebra $V = \bigoplus_{n\geq 0} V_n$ one may associate an algebra structure on V_2, and if $\dim V_0 = 1$ and $V_1 = 0$, then this algebra becomes commutative (called the Griess algebra of V).

For every $r \in \mathbb{C}$ and integer $d \geq 2$, T. Ashihara and M. Miyamoto [J. Algebra **321** (2009), no. 6, 1593–1599; MR2498258 (2009k:17045)] constructed a vertex operator algebra of central charge dr, whose Griess algebra is the simple Jordan algebra of symmetric $d \times d$ matrices. Here it is proved that this vertex operator algebra is simple if and only if r is not an integer, and generators of its maximal proper ideal are constructed.

Reviewed by *Pasha Zusmanovich*

MR2586982 (2011b:16099) 16S37
Cassidy, Thomas (1-BCNL)
Quadratic algebras with Ext algebras generated in two degrees. (English summary)
J. Pure Appl. Algebra **214** (2010), *no. 7,* 1011–1016.

A nonnegatively-graded associative algebra A over a field K is called Koszul if its bigraded Ext algebra $\bigoplus_{i\leq j} \mathrm{Ext}_A^{ij}(K,K)$ (the first degree is the cohomology degree, the second degree comes from the grading of the algebra) is generated by $\mathrm{Ext}_A^{11}(K,K)$. A quadratic algebra is Koszul if and only if $\mathrm{Ext}_A^{ij}(K,K) = 0$ for all $i < j$. This, nowadays classical, notion admits various generalizations by allowing the Ext algebra to be generated in few, but more than one, bidegrees, or by relaxing the vanishing condition of Ext's.

In this paper, for every integer $m \geq 3$, an algebra is constructed such that its Ext algebra is generated in bidegrees $(1,1)$ and $(m, m+1)$. For such algebras, $\mathrm{Ext}_A^{ij}(K,K) = 0$ for all $i < j \leq m$.

Reviewed by *Pasha Zusmanovich*

MR2603876 (2011d:17007) 17A99 (16S99 17A30)

Pozhidaev, A. P. (RS-AOSSI)

0-dialgebras with bar-unity and nonassociative Rota-Baxter algebras. (Russian. Russian summary)

Sibirsk. Mat. Zh. **50** (2009), *no. 6,* 1356–1369; *translation in Sib. Math. J.* **50** (2009), *no. 6,* 1070–1080.

The paper under review touches upon several questions related to dialgebras introduced by J.-L. Loday [in *Dialgebras and related operads*, 7–66, Lecture Notes in Math., 1763, Springer, Berlin, 2001; MR1860994 (2002i:17004)], i.e. linear spaces equipped with two bilinear products, $\vdash$ and $\dashv$, satisfying various additional axioms.

According to P. Kolesnikov [Sibirsk. Mat. Zh. **49** (2008), no. 2, 322–339; MR2419658 (2009b:17002)], a 0-dialgebra is a dialgebra satisfying two axioms $(x \dashv y) \vdash z = (x \vdash y) \vdash z$ and $z \dashv (x \vdash y) = z \dashv (x \dashv y)$. If, additionally, a dialgebra satisfies the third axiom $(x \vdash y) \dashv z = x \vdash (y \dashv z)$, then it is called associative. A bar unity in a 0-dialgebra is a (not necessarily unique) element e satisfying some natural axioms generalizing the axioms of unity in an ordinary algebra.

For a 0-dialgebra with a bar unity e, define new operations $x \star y$ and $R(x)$ as linear combinations with arbitrary constant coefficients of the terms $\dot{x}y$, $x\dot{y}$, $\dot{y}x$, $y\dot{x}$, and the terms x, $x\dot{e}$, $\dot{e}x$, respectively. Here expressions like $\dot{x}$ denote the middle of a dialgebraic monomial as defined by Loday [op. cit.].

First, it is determined, via straightforward computations, when the so-defined operations $\star$ and R define a structure of a Rota–Baxter algebra, i.e. when the following relation holds: $R(x) \star R(y) = R(R(x) \star y + x \star R(y) + \lambda x \star y)$ for some element λ from the ground field. This result may be utilized in defining a structure of a Rota–Baxter algebra on a unital conformal algebra.

Second, it is proved that an associative 0-dialgebra embeds into an associative 0-dialgebra with a bar unity.

Third, it is proved that the definition of a variety of dialgebras given by Kolesnikov [op. cit.] is equivalent to another definition, given in terms of bimodules.

There seems to be a slight confusion in terminology: while in §2 "dialgebra" means an "associative 0-dialgebra", in §3 it is apparently used in the most general meaning (adopted in this review), i.e. "a linear space equipped with two bilinear products".

Reviewed by *Pasha Zusmanovich*

MR2588639 (2011a:17019) 17B22 (17B08 20G40)

Ignat′ev, M. V. (RS-SAMA)

Orthogonal subsets of classical root systems and coadjoint orbits of unipotent groups. (Russian. Russian summary)

Mat. Zametki **86** (2009), *no. 1,* 65–80; *translation in Math. Notes* **86** (2009), *no. 1-2,* 65–80.

Let U be the maximal unipotent subgroup of the classical matrix group of type B, C or D over a finite field. In this paper, dimensions of orbits of the coadjoint action of U on $\mathrm{Lie}(U)^*$, associated with a set of pairwise orthogonal roots, are expressed in terms of the corresponding Weyl group. As a consequence, dimensions of irreducible representations of U are obtained.

The corresponding results for groups of type A were obtained earlier by A. N. Panov [J. Math. Sci. **151** (2008), no. 3, 3018-3031].

Reviewed by *Pasha Zusmanovich*

MR2568398 (2010m:17009) 17B22 (17A35 17B67 20F55)

Feingold, Alex J. (1-SUNY2); **Kleinschmidt, Axel [Kleinschmidt, Axel[2]]** (B-ULB-TPM); **Nicolai, Hermann** (D-MPIGP)

Hyperbolic Weyl groups and the four normed division algebras. (English summary)

Vertex operator algebras and related areas, 53–64, *Contemp. Math.*, 497, *Amer. Math. Soc.*, *Providence, RI*, 2009.

The Weyl group is a group of symmetries of the root system of a split finite-dimensional simple Lie algebra $\mathfrak{g}$. Replacing in this classical notion the Lie algebra $\mathfrak{g}$ by various generalizations, one arrives at various generalizations of the Weyl group.

This paper considers the situation when $\mathfrak{g}$ is a hyperbolic (or almost affine in another terminology [see D. Chapovalov et al., "The classification of almost affine (hyperbolic) Lie superalgebras", preprint, arxiv.org/abs/0906.1860]) Kac-Moody algebra, and the structure of the corresponding hyperbolic Weyl groups is described as discrete subgroups of matrix groups over the four real normed division algebras.

This is a short account without proofs; full details can be found in another paper by the authors [J. Algebra **322** (2009), no. 4, 1295–1339; MR2537656].

The paper is nicely illustrated by pictures of "important species and interesting individuals" (quoting C. L. Siegel): Geoffrey Mason (in whose Festschrift the paper appeared), the Poincaré disc model of hyperbolic plane, and the Fano plane.

{For the entire collection see MR2568393 (2010e:17001)}

Reviewed by *Pasha Zusmanovich*

MR2549450 (2010j:16022) 16E40 (16E05 16S34)

Generalov, A. I. (RS-STPTMM)

Hochschild cohomology of algebras of semidihedral type. I. Group algebras of semidihedral groups. (Russian. Russian summary)

Algebra i Analiz **21** (2009), *no. 2,* 1–51; *translation in St. Petersburg Math. J.* **21** (2010), *no. 2,* 163–201.

The goal of this paper is to compute the Hochschild cohomology, with coefficients in itself, of a class of associative algebras R generated by two elements X,Y subject to the relations

$$X^2 = Y(XY)^{k-1}, \quad Y^2 = 0, \quad (XY)^k = (YK)^k, \quad X(YX)^k = 0$$

for some integer $k \geq 2$. These algebras are finite-dimensional, include group algebras of semidihedral groups of order 2^n (for $k = 2^{n-2}$), and appear in the classification of group blocks of tame representation type [K. Erdmann, *Blocks of tame representation type and related algebras*, Lecture Notes in Math., 1428, Springer, Berlin, 1990; MR1064107 (91c:20016)].

A certain double complex is constructed, whose total complex is proved to be a minimal projective resolution of the enveloping algebra $R \otimes R^{\mathrm{op}}$.

Reviewed by *Pasha Zusmanovich* (Kópavogur)

MR2543503 (2010j:16024) 16E40 (17A32 17B35 17B56 19D55)

Lodder, Jerry M. [Lodder, Gerald M.] (1-NMS)

A comparison of Leibniz and cyclic homologies. (English summary)

Comm. Algebra **37** (2009), *no. 8,* 2557–2569.

The author notes the relationship between the following six homology theories associated with a Lie algebra L and its universal envelope $U(L)$: Leibniz and Chevalley-Eilenberg homology of L and a Lie algebra associated with $U(L)$, and Hochschild and cyclic homology of $U(L)$. Specializing to the Lie algebra of polynomial vector fields W_1, he proves that the Godbillon-Vey cocycle, which generates the third homology of W_1, represents a nonzero homology class in all these six theories.

Reviewed by *Pasha Zusmanovich*

MR2483274 (2010b:16015) 16E40 (18G35)

Carboni, Graciela (RA-UBA-CBC); **Guccione, Jorge A.** (RA-UBAS);
Guccione, Juan J. (RA-UBAS)

Cyclic homology of monogenic extensions in the noncommutative setting. (English summary)

J. Algebra **321** (2009), *no. 2,* 404–428.

This paper investigates the Hochschild and cyclic homology of associative algebras which could be considered as a noncommutative analogue of algebras of the form $K[x]/(f)$, where $f \in K[x]$ is a monic polynomial, and related algebras such as rank 1 Hopf algebras, generalizing and unifying the previous works, including those of the authors, dedicated to the commutative case.

The homological perturbation lemma is used to express the homology of algebras such as the homology of a certain mixed complex (roughly, a double complex consisting of cochain and chain complexes with compatible differentials), a ubiquitous construction appearing in cyclic homology.

Reviewed by *Pasha Zusmanovich* (Kópavogur)