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MATHEMATICAL LIFE

Askar Serkulovich Dzhumadil'daev (on his 60th birthday)

Askar Serkulovich Dzhumadil'daev, academician of the National Academy of Sciences of the Republic of Kazakhstan, professor, and doctor of the physical and mathematical sciences, was born on 25 February 1956 at the railway station Chiili in Kzyl-Orda oblast. His great-grandfather Shon was a man of high position, who served as a volost' (district) administrator. He owned a large wooden house, which was unusual for those localities; in fact, he kept a caravansary (a roadside inn). During the collectivisation all the properties were confiscated, and Shon's children were subjected to political repression. One of his sons, Dzhumadil'da, narrowly escaped by leaving the house one day before he would have been arrested, and fled to Uzbekistan with his children. After Stalin's death they returned to their home village. Askar's father Serkul had finished four years of school. Askar's maternal grandfather Usen worked for forty years as a trackman on the railroad and was awarded the Order of Lenin. The harsh climate of the 27th junction, where he lived, corresponded well to the original name of the place: 'Boroinak' ('Wolf lair'). There were no schools there, and so Askar's mother Kul'shat remained illiterate. Communication with the outside world was via passing trains. Chingiz Aitmatov's novel "Blizzard junction" contains an excellent depiction of the life of a railroad trackman, similar to the life in Boroinak. Although Askar's parents were not well educated, there was a cult of knowledge in the family. His grandfather encouraged his passion for books. Askar was keen on poetry and history, and he wrote articles for *the Kazakhstan Pioneer*.[1] In the 6th form he was sent to Kzyl-Orda to participate in the regional mathematical olympiad. Back then the winners of regional olympiads were sent directly to the All-USSR Olympiad. A week-long trip on the train from Chiili to Kiev via Moscow made a strong impression on Askar. In Moscow he saw that books can not only

[1] A newspaper for schoolchildren.

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be fiction but can also be devoted to mathematics. He bought a lot of books on mathematics and decided that mathematics is a worthy occupation, since it makes it possible to go to big cities and buy interesting books.

In 1972 Askar applied for admission to the Faculty of Mechanics and Mathematics at Moscow State University. As a graduate of a national school, he was allowed to write an exposition in Russian (on some piece of literature) rather than an original composition, as part of the entrance examinations. The exposition was based on a fragment of Tolstoy's novel *War and Peace*. His attempt to write in the style of Tolstoy resulted in a catastrophe — he failed the examination in the Russian language. On his way to the Entrance Commission, located on the 12th floor, he met in the lift a person whom he had seen at various olympiads. This man also recognised an olympiad winner in the boy, and after learning about the reason for his sadness, advised him to appeal the decision and helped him write the appeal application. A miracle happened: of the dozens of applications his was one of those supported. Askar drew two conclusions: 1) if you are not Lev Nikolaevich Tolstoy, do not write in compound sentences; 2) follow the advice of al-Khwarizmi: try to turn minuses into pluses, drawbacks into advantages.

Higher algebra was taught by A. I. Kostrikin. From his lectures Askar realised that the most comfortable object for him was a field of characteristic $p > 0$, especially a field of characteristic 2, where there was no need to worry about signs. In accordance with rule 2), he transformed his poor knowledge of Russian into a strong weapon. He made notes of the lectures twice: first in the auditorium, and then at home, by rewriting the notes with a dictionary. In particular, in this way he overcame his fear of the History of the Communist Party of the Soviet Union. This subject, which used to terrorise Soviet students, taught him to read the works of the classics — both the founders of dialectical materialism and the classics of Russian literature and mathematics.

Dzhumadil'daev defended his Ph.D. thesis in 1981, in the Steklov Mathematical Institute (his advisor was Kostrikin), and his D.Sc. thesis in 1988, in the Leningrad Branch of the Institute. The dissertations were devoted to the cohomology of Lie algebras of positive characteristic. His research interests are diverse: the theory of Lie algebras, the theory of non-associative algebras, and combinatorics. We consider in detail some of his results.

Cohomology and extensions. The foundations of the theory of Lie algebras over a field of characteristic zero include the Levi–Malcev theorem on the splitting of the radical. This theorem, in turn, follows from Whitehead's lemma, which states that the cohomology groups of semisimple Lie algebras with coefficients in an irreducible module are trivial if the module is non-trivial. Whitehead's lemma does not hold in the case of characteristic $p > 0$. Jacobson proved that any modular Lie algebra has an indecomposable module. Seligman stated the conjecture that for any $0 \leqslant k \leqslant n$ each modular Lie algebra of dimension n has a module with non-trivial k-cohomology. Dzhumadil'daev proved Seligman's conjecture. Namely, he established [1], [2] that for any finite-dimensional Lie algebra of characteristic $p > 0$:

(a) there exists a module with non-trivial cohomology;

(b) an irreducible module with non-zero cohomology is restricted, and the number of such modules is finite;

(c) for any cocycle with values in a finite-dimensional module there exists a finite-dimensional extension of this module in which the cocycle becomes a coboundary.

In the one- and two-dimensional cases these results give the following modular analogues of the Levi–Malcev theorems:

(i) any finite-dimensional Lie algebra of characteristic p has a non-split extension;

(ii) the number of such extensions by irreducible modules is finite;

(iii) any non-split extension by a soluble ideal is equivalent to a subextension of a split extension, and any module is equivalent to a submodule (quotient module) of some indecomposable module.

Dzhumadil'daev observed that these results have infinite-dimensional analogues in the case of characteristic 0. As an application, he constructed all central extensions of the Lie algebras of Cartan types and of the Lie algebras of pseudodifferential operators. He also calculated the non-split extensions of the Lie algebras of Cartan types by irreducible modules [3]–[5]. In [6] he developed the cohomology theory of right-symmetric algebras.

Generalized commutators and identities. For a variety of algebras $\mathscr{C}$, consider the class $\mathscr{C}^{(q)}$ of algebras in $\mathscr{C}$ with multiplication given by the q-commutator: $a \circ_q b = a \circ b + q\, b \circ a$. It turns out that for $q^2 \neq 1$ the class $\mathscr{C}^{(q)}$ is also a variety, and moreover, the varieties $\mathscr{C}$ and $\mathscr{C}^{(q)}$ are isomorphic as categories. This means that from the category viewpoint interesting classes of algebras of the form $\mathscr{C}^{(q)}$ can arise only in the study of Lie and Jordan commutators [12].

More than hundred years ago Sophus Lie observed that a composition of vector fields $X_1 = a_i\partial_i$ and $X_2 = b_j\partial_j$ is not necessarily a vector field, but their commutator

$$[X_1, X_2] = a_i\, \partial_i(b_j)\, \partial_j - b_j\, \partial_j(a_i)\, \partial_i$$

is a vector field. This observation lies in the foundation of the theory of Lie algebras and Lie groups. In [10] it was proved that Lie's construction can be generalized to the N-ary case:

$$[X_1, \dots, X_N] = \sum_{\sigma \in \mathrm{Sym}_N} (\operatorname{sign} \sigma) X_{\sigma(1)} \cdots X_{\sigma(N)} \in \mathrm{Vect}(n)$$

if $N = n^2 + 2n - 2$. Thus, it was established that along with the Lie commutator the space of vector fields on an n-dimensional manifold admits new tensor operations [15], [16].

Novikov algebras and their generalizations [7], [8], [14]. Novikov algebras are defined by the identity of right-symmetricity of the associator and the identity of left-commutativity. The basic example of a Novikov algebra is the algebra of polynomials with respect to the multiplication $a \circ b = a'b$, where a' is the derivative. Dzhumadil'daev started studying the identities of polynomial algebras with respect to multiplications constructed by using the operations of derivation and integration. He constructed a number of new classes of algebras generalizing Novikov algebras. It turned out that such algebras have many non-equivalent identities, but if the relation of weak equivalence is taken into account, then the picture becomes fairly treatable and interesting. For example, the space of polynomials with respect to multiplications generated by differential operators of degree 1, that is, multiplications of the form $a \star b = \alpha a'b + \beta ab'$, generate three classes of algebras: Witt algebras ($a \star b = a'b - ab'$), Novikov algebras ($a \star b = a'b$), and algebras with the multiplication $a \star b = (ab)'$. The last algebra satisfies an identity of degree 4:

$$(a \star b) \star (c \star d) - (a \star d) \star (c \star b) = (a, b, c) \star d - (a, d, c) \star b.$$

Dzhumadil'daev was elected a member of the Supreme Soviet (Council) of the Republic of Kazakhstan of the 12th (1991–1993) and 13th (1993–1995) convocation sessions and was a Deputy Chairman of the Committee on Science and Education of the parliament. One of the points of his electoral platform was a proposal for creating a special fund to support gifted young people. In the framework of the programme "Bolashak" ("Future"), more than ten thousand Kazakhstani students have been given the opportunity to continue their education in prestigious universities abroad. Dzhumadil'daev took part in the passage of historic resolutions in the parliament, such as the law about the independence of Kazakhstan and the first constitution of the Republic of Kazakhstan. In 1995–1996 he was a research fellow of the Alexander von Humboldt Foundation, in 2011 he received the State Prize of the Republic of Kazakhstan, and in 2012 the Khwarizmi International Award.

Askar Serkulovich is a superb teacher; he has passed on his enthusiasm and his love for mathematics to newer and newer generations of students. By the results of students' questionnaires he was three times voted the best professor of the Kazakh-British Technical University.

We wish Askar Serkulovich robust health, long life, and new accomplishments.

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