PERIODIC QUANTUM GRAPHS WITH PREDEFINED SPECTRAL GAPS

ANDRII KHRABUSTOVSKYI

The name "quantum graph" is used for a pair (Γ, \mathcal{H}) , where Γ is a networkshaped structure of vertices connected by edges ("metric graph") and \mathcal{H} is a second order self-adjoint differential operator ("Hamiltonian") on it, which is determined by differential operations on the edges and certain interface conditions at the vertices. Quantum graphs arise naturally in mathematics, physics, chemistry and engineering as models of wave propagation in quasione-dimensional systems looking like a narrow neighbourhood of a graph. We refer to the book [2], which is a nice introduction to the topic.

In many applications (for example, to graphen and carbon nano-structures) periodic quantum graphs are studied. It is known that the spectrum of the corresponding Hamiltonians has a band structure, i.e. it is a locally finite union of compact intervals called *bands*. In general the neighbouring bands may overlap. A bounded open interval is called a *gap* if it has an empty intersection with the spectrum, but its edges belong to it. In general the presence of gaps in the spectrum is not guaranteed. Existence of spectral gaps is important because of various applications, for example in physics of photonic crystals.

The current research concerns spectral properties of a class of periodic quantum graphs. The main peculiarity of the graphs under investigation is that their spectral gaps can be nicely controlled via a suitable choice of the graph geometry and of coupling constants involved in interface conditions at its vertices. The results are published in [1]

This is a joint work with Diana Barseghyan (University of Ostrava).

References

- [1] D. Barseghyan, A. Khrabustovskyi, Gaps in the spectrum of a periodic quantum graph with periodically distributed δ' -type interactions, Journal of Physics A: Mathematical and Theoretical, 48(25) (2015), 255201
- [2] G. Berkolaiko, P. Kuchment, Introduction to quantum graphs, American Mathematical Society, Providence, RI, 2013.

DEPARTMENT OF MATHEMATICS, KARLSRUHE INSTITUTE OF TECHNOLOGY, GERMANY