

LEAKY CONICAL SURFACES: SPECTRAL ASYMPTOTICS, ISOPERIMETRIC PROPERTIES, AND BEYOND

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In the recent time there is a considerable growth of interest to spectral problems on unbounded conical domains. One of the common features shared by many such problems is infiniteness of the discrete spectrum below the threshold of the essential spectrum. In [1] we considered three-dimensional Schrödinger operator

$$H_C = -\Delta - \alpha\delta(\cdot - C)$$

with attractive δ -interaction of strength $\alpha > 0$ supported on the circular unbounded conical surface $C = \{(x, y, z) \in \mathbb{R}^3 : z = \cot(\theta)\sqrt{x^2 + y^2}\}$, $\theta \in (0, \pi/2)$. The Hamiltonian H_C can be defined as a self-adjoint operator in $L^2(\mathbb{R}^3)$ and it models charged particle attracted by the (leaky) surface C . It is shown in [1] that $\sigma_{\text{ess}}(H_C) = [-\alpha^2/4, +\infty)$ and that the discrete spectrum of H_C is infinite. In this talk, we present recent results on the asymptotics of the counting function $\mathcal{N}_{-\alpha^2/4-E}$ for the discrete spectrum of H_C , obtained in [3]. It turns out that the counting function for H_C exhibits the following asymptotic behaviour

$$\mathcal{N}_{-\alpha^2/4-E} \sim \frac{\cot \theta}{4\pi} |\ln E|, \quad E \rightarrow 0+.$$

Moreover, we will discuss spectral properties of Schrödinger operators with δ -interactions supported on more general (not necessarily circular) cones, considered in [2]. In this context, our main aim is to show that in a certain reasonable class of cones the circular cone maximises the ground state eigenvalue. Besides that, several related questions will be highlighted more briefly, one of them is connected to a generalization of this spectral problem to higher space dimensions and another one to spectral properties of δ -interactions supported on truncated cones.

REFERENCES

- [1] J. Behrndt, P. Exner, and V. Lotoreichik, Schrödinger operators with δ -interactions supported on conical surfaces, *J. Phys. A* **47** (2014), 355202.
- [2] P. Exner and V. Lotoreichik, A spectral isoperimetric inequality for cones, *submitted*, *arXiv:1512.01970*.
- [3] V. Lotoreichik and T. Ourmières-Bonafos, On the bound states of Schrödinger operators with δ -interactions on conical surfaces, *to appear in Comm. Partial Differential Equations*, *arXiv:1510.05623*.