GEOMETRIC STRUCTURES ON LIE GROUPS AND POST-LIE ALGEBRAS

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Pre-Lie algebras have been introduced in 1963 independently by Vinberg, in the context of convex homogeneous cones, and by Gerstenhaber, in the context of Hochschild cohomology and deformation theory of algebras. Since then they have appeared in many other areas, e.g., in differential geometry, operad theory, control theory, quantum field theory (Connes-Kreimer quantizations) and differential equations. A natural generalization is the concept of a *Post-Lie algebra*. Such algebras have been introduced in 2007 by Vallette in connection with homology of partition posets and Koszul operads. They independently arose in our studies on nil-affine actions of Lie groups in 2012, and in other areas, e.g., in the analysis of flows on manifolds and homogeneous spaces.
One of the main problems motivated by differential geometry is the *existence* of pre-Lie algebra structures on a given Lie algebra, repectively of post-Lie algebra structures on a given pair $(\mathfrak{g}, \mathfrak{n})$ of Lie algebras. This corresponds to the existence question of left-invariant affinely-flat structures on Lie groups (Milnor 1977), and to the question which Lie groups G can act on Lie groups N by simply-transitive nil-affine actions. In joint work with Karel Dekimpe and Wolfgang Moens we have obtained several algebraic results concerning these questions. For *commutative* post-Lie algebra structures we have the most powerful structure results.

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