On the constant astigmatism equation and surfaces of constant astigmatism

Adam Hlaváč, Michal Marvan

Mathematical Institute in Opava, Na Rybníčku 1, 746 01 Opava, Czech Republic

Adam.Hlavac@math.slu.cz, Michal.Marvan@math.slu.cz

Surfaces of constant astigmatism, i.e. surfaces characterized by the condition $\rho_2 - \rho_1 = \text{const} \neq 0$, where ρ_1, ρ_2 are the principal radii of curvature, were already known at the end of the nineteenth century. Their evolutes are pseudospherical surfaces, which, themselves, correspond to solutions of the sine-Gordon equation $u_{\xi\eta} = \sin u$.

In 2009, after a century in oblivion, the subject of constant astigmatism surfaces was resurrected by H. Baran and M. Marvan in their work [1] concerning the systematic search for integrable classes of Weingarten surfaces. In particular, it was shown that surfaces of constant astigmatism, parameterized by adapted curvature coordinates x, y, correspond to solutions of the integrable equation

$$z_{yy} + \left(\frac{1}{z}\right)_{xx} + 2 = 0$$

called the *constant astigmatism equation (CAE).* Solutions of the CAE can be alternatively interpreted as spherical *orthogonal equiareal patterns*, with relevance to two-dimensional plasticity [2].

In [4] we presented a nonlinear superposition formula, based on the Bäcklund transformation for the sine-Gordon equation, and producing infinitely many exact solutions of the CAE from a given seed. Moreover, one can routinely construct corresponding constant astigmatism surfaces. The simplest solution of the CAE, one can use as a seed, is the von Lilienthal one, $z = 1/(1 - x^2)$, which corresponds to zero solution of the sine-Gordon equation. The hierarchy of solutions, arising from this seed, is called *multisoliton solutions* of the CAE.

Recently, we have successfully constructed and planted another seed, namely the *Lipschitz solution* studied in [3]. A new infinite hierarchy of solutions of the CAE together with corresponding surfaces of constant astigmatism then follows by routine algebraic manipulations and differentiation.

In the talk, an overview of known exact solutions of the CAE will be given. Corresponding surfaces of constant astigmatism will be shown as well.

References

[1] H. Baran and M. Marvan, On integrability of Weingarten surfaces: a forgotten class, *J. Phys. A: Math. Theor.* **42** (2009) 404007.

[2] A. Hlaváč and M. Marvan, Another integrable case in two-dimensional plasticity, *J. Phys. A: Math. Theor.* **46** (2013) 045203.

[3] A. Hlaváč and M. Marvan, On Lipschitz solutions of the constant astigmatism equation, *Journal of Geometry and Physics* **85**, 88–98 (2014), 10.1016/j.geomphys.2014.05.020.

[4] A. Hlaváč, On multisoliton solutions of the constant astigmatism equation, *J. Phys. A: Math. Theor.* **48** (2015) 365202.