SPECTRAL OPTIMIZATION FOR THE ROBIN LAPLACIAN ON EXTERIOR DOMAINS

VLADIMIR LOTOREICHIK
CZECH ACADEMY OF SCIENCES

The exterior domain $\Omega^{\rm ext}$ is defined as the complement of a bounded domain $\Omega \subset \mathbb{R}^{\mathsf{d}}$ in the Euclidean space $\mathbb{R}^{\mathsf{d}}$, $\mathsf{d} \geqslant 2$. The domain $\Omega^{\rm ext}$ is unbounded, but having a compact boundary. We will discuss shape optimization for the Robin Laplacian on $\Omega^{\rm ext}$. The setting of unbounded domains is much less understood in the spectral shape optimization. Moreover, there is certain interplay with the continuous spectrum, which leads to optimization of somewhat unusual spectral quantities.

Typically, we attempt to prove that the exterior of a ball maximizes the lowest Robin eigenvalue of $\Omega^{\rm ext}$ under different constraints on Ω being imposed. The choice of these constraints significantly depends on the dimension. In two dimensions ($\mathsf{d} = 2$), the number of connected components in Ω enters the constraint. In higher dimensions ($\mathsf{d} \geqslant 3$), we obtain a result for convex Ω only, under the constraint involving a Willmore-type energy, which was originally introduced in differential geometry to measure the "roundedness" of a surface. These results are obtained in a joint work with D. Krejčiřík.