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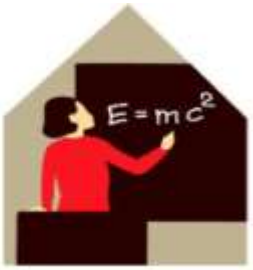


# On the classical A. Buhl problem, its G. Pfeiffer solution and its connection with completely integrable heavenly type dynamical systems

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## ABSTRACT

We will review a novel Lie-algebraic approach to studying integrable heavenly type multi-dimensional dynamical systems and its relationships to old and recent investigations of the classical M. A. Buhl problem of describing compatible linear vector field equations by a prominent ukrainian mathematician G. Pfeiffer and in modern times by members of the japanese Sato school. Eespecially we analyze the related Lie-algebraic structures and integrability properties of a very interesting class of nonlinear dynamical systems called the dispersionless heavenly type equations, which were initiated by Plebański and later analyzed in a series of articles. The AKS-algebraic and related $R$-structure schemes are used to study the orbits of the corresponding co-adjoint actions, which are intimately related to the classical Lie–Poisson structures on them.

It is demonstrated that their compatibility condition coincides with the corresponding heavenly type equations under consideration. It is shown that all these equations originate in this way and can be represented as a Lax compatibility condition for specially constructed loop vector fields on the torus. The infinite hierarchy of conservations laws related to the heavenly equations is described, and its analytical structure connected with the Casimir invariants, is mentioned. In addition, typical examples of such equations, demonstrating in detail their integrability via the scheme devised herein, are presented. The relationship of a very interesting Lagrange–d'Alembert type mechanical interpretation of the devised integrability scheme with the Lax–Sato equations is also discussed.