The algebraic and geometric classification of nilpotent (binary, mono) Leibniz algebras

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Introduction

One of the classical problems in the theory of non-associative algebras is to classify (up to isomorphism) the algebras of dimension n from a certain variety defined by some family of polynomial identities. It is typical to focus on small dimensions, and there are two main directions for the classification: algebraic and geometric. Varieties as Jordan, Lie, Leibniz or Zinbiel algebras have been studied from these two approaches.

The algebraic classification (up to isomorphism) of n-dimensional algebras from a certain variety defined by a certain family of polynomial identities is a classic problem in the theory of non-associative algebras. There are many papers devoted to algebraic classification of small-dimensional algebras in several varieties of associative and non-associative algebras.

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Articles on classification of Leibniz algebra.

1. Ayupov Sh.A., Omirov B.A. (1999). On 3-dimensional Leibniz algebras.
2. Rikhsiboev, I. M., Rakhimov, I. S. (2012). Classification of three dimensional complex Leibniz algebras.
3. Casas J.M., Insua M.A., Ladra M., Ladra S. (2012). An algorithm for the classification of 3-dimensional complex Leibniz algebras.
4. Albeverio, S., Omirov, B. A., Rakhimov I. S. (2006). Classification of 4-dimensional nilpotent complex Leibniz algebras.
5. Demir I., Misra K.C., Stitzinger E. (2016). On classification of four-dimensional nilpotent Leibniz algebras.
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Binary and mono Leibniz algebras

Let Ω denote the algebras defined by a family of polynomial identities, then we say that an algebra $A \in \Omega_i$ if and only if each i-generated subalgebra of A gives an algebra from Ω. In particular, if $A \in \Omega_1$, then A is a mono-Ω algebra, if $A \in \Omega_2$, then A is a binary-Ω algebra.

For example, let Ass be the class of associative algebras, then by Artin's theorem, the class Ass_2 coincides with the class of alternative algebras. Albert's theorem follows that the class Ass_1 coincides with the class of power-associative algebras.

It is easy to see that Lie_1 coincides with anticommutative algebras, i.e., they satisfy the identity $x^2 = 0$. The algebraic theory of binary Lie algebras was developed in some papers by Kuzmin, Filippov, and Grishkov. So, Kuzmin proved Engel's theorem for binary Lie algebras. Recently, defining identities for mono and binary Zinbiel algebras have been described[1].

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A concept of compatible algebras is the sum of two algebras belonging to Ω. Multiplications of both algebras is Ω and the sum of those algebras has an Ω multiplication as well. For example, an associative compatible algebra is two multiplications - each multiplication is associative and their sums also give an associative multiplication. So here's the question:

1. take a variety of algebras - look at their compatible algebras - construct binary ones of this variety;
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Binary and mono Leibniz algebras

Below we introduce the notations.

$$\mathcal{J}(x,y,z) = [[x,y],z] + [[y,z],x] + [[z,x],y]$$
$$\mathcal{L}(x,y,z) = (xy)z - x(yz) + y(xz).$$

Definition

Let $(A,[-,-])$ be an anticommutative algebra. Then $(A,[-,-])$ is a Malcev algebra if the following is true

$$\mathcal{J}(x,y,[x,z]) = [\mathcal{J}(x,y,z),x].$$

Definition

A complex vector space is called a binary Leibniz (binary Lie) algebra if every two-generated subalgebra is a Leibniz (Lie) algebra. A complex vector space is called a mono Leibniz (mono Lie) algebra if every one-generated subalgebra is a Leibniz (Lie) algebra.

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In the work of A.T. Gainov [2] it is proved that the algebra $\mathcal{A}$ is binary Lie if and only if it holds the identities

$$[x, x] = 0, \quad \mathcal{J}(x, y, [x, y]) = 0.$$

Every Lie algebra is a Malcev algebra and every Malcev algebra is a binary Lie algebra. Since the Leibniz algebras are noncommutative generalizations of Lie algebras, it follows that every binary Lie algebra is a binary Leibniz algebra. Every Leibniz algebra is a binary Leibniz algebra and every binary Leibniz algebra is a mono Leibniz algebra. Moreover, every mono Lie algebra is a mono Leibniz algebra.

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The inclusion diagram looks as follows [3]:

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\end{array}$$

Here, an unknown algebraic variety was studied by Dzhumadil'daev, who looked for algebras that satisfy several conditions, but could not find them.

Kazin and Yeskendir found an algebra defined by the following identities:

$$\mathcal{L}(x, y, xz) - \mathcal{L}(x, y, z)x = 0.$$

This algebra was called the $\mathcal{N}$ algebra. They also showed that this relation holds for $\mathcal{N}$ algebras.

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Examples

Example 1. Non-$\mathcal{N}$, but Binary Leibniz algebra:

$$e_1e_2 = e_4, \quad e_1e_3 = e_1, \quad e_2e_3 = e_2.$$

Example 2. Non Leibniz but a $\mathcal{N}$ algebra:

$$e_1e_2 = -e_1, \quad e_1e_3 = e_4, \quad e_4e_2 = e_4, \quad e_3e_2 = -e_3.$$

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Ismailov N., Dzhumadil'daev A.[4] proved that the algebra **A** is binary Leibniz if and only if it satisfies the identities

$$\begin{array}{rclrcl} \mathcal{L}(x,y,z)+\mathcal{L}(y,x,z) & = & 0, & \mathcal{L}(x,y,z)+\mathcal{L}(z,y,x) & = & 0, \\ \multicolumn{4}{r}{\mathcal{L}(x,y,zt)+\mathcal{L}(x,t,zy)+\mathcal{L}(z,y,xt)+\mathcal{L}(z,t,xy)} & = & 0. \end{array}$$

The algebra **A** is mono Leibniz if and only if it satisfies the identities [5]

$$\mathcal{L}(a,a,a) = 0, \quad \mathcal{L}(aa,a,a) = 0.$$

By using linearization for these identities

$$\mathcal{L}(x,y,z)+\mathcal{L}(y,x,z)+\mathcal{L}(y,z,x)+\mathcal{L}(x,z,y)+\mathcal{L}(z,x,y)+\mathcal{L}(z,y,x)=0,$$
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[4] Ismailov N., Dzhumadil'daev A., Mathematical Notes, (2021)

[5] Gainov A.T., Algebra Logic, (2010)

Ismailov N., Dzhumadil'daev A.[4] proved that the algebra **A** is binary Leibniz if and only if it satisfies the identities

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The algebra **A** is mono Leibniz if and only if it satisfies the identities [5]

$$\mathcal{L}(a,a,a) = 0, \quad \mathcal{L}(aa,a,a) = 0.$$

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Binary and mono Leibniz algebras

From the definition of binary Leibniz algebras we can conclude the following:

- There are no nontrivial 1-dimensional nilpotent binary Leibniz (Mono Leibniz) algebras.
- Two-dimensional and three-dimensional nilpotent binary Leibniz (Mono Leibniz) algebras are Leibniz algebras.
- Two-generated binary Leibniz algebras are Leibniz algebra.
- A binary Leibniz (Mono Leibniz) algebra $\mathfrak{L}$, such that for $\mathfrak{L}^3 = 0$, is a Leibniz algebra.

Thus, non-Leibniz binary Leibniz algebras should be at least three generated. Consequently, we have that any nilpotent binary Leibniz algebra with a dimension less than five is a Leibniz algebra.

Thus, we conclude that any nilpotent non-Leibniz mono Leibniz algebra has at least two generators and $\mathfrak{L}^3 \neq 0$. Consequently, we have that any nilpotent mono Leibniz algebra with a dimension less than four is a Leibniz algebra.

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Classification theorem for 5-dimensional nilpotent Leibniz algebras

The algebraic classification of complex 5-dimensional nilpotent Leibniz algebras consists of three parts:

1. 5-dimensional algebras with identity $xyz = 0$ (also known as 2-step nilpotent algebras) are the intersections all varieties of algebras defined by a family of polynomial identities of degree three or more; for example, they are in the intersection of associative, Zinbiel, Leibniz, etc, algebras. All these algebras can be obtained as central extensions of zero-product algebras. (Geometric classification [6].)

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2. 5-dimensional nilpotent symmetric Leibniz (non-2-step nilpotent) algebras, which are central extensions of nilpotent Lie algebras with non-zero product of a smaller dimension, are given in [7].
3. 5-dimensional nilpotent non-symmetric Leibniz algebras are given above and summarized in Theorem (1) (see below).

Theorem (1)

Up to isomorphism, there are infinitely many isomorphism classes of complex 5-dimensional nilpotent (non-symmetric) Leibniz algebras, described explicitly in terms of 2 *two-parameter families* 18 *one-parameter families and* 62 *additional isomorphism classes.*

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Theorem (2)

Let **B** *be a complex* 5*-dimensional nilpotent binary Leibniz algebra. Then* **B** *is a Leibniz algebra or isomorphic to one algebra from the following list:*

$\mathbf{B}_{01}$	:	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_3 = -e_5$	
$\mathbf{B}_{02}$	:	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_3 = -e_5$	$e_4e_4 = e_5$
$\mathbf{B}_{03}$	:	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_1 = e_5$	$e_4e_3 = -e_5$
$\mathbf{B}_{04}$	:	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_1 = e_5$	$e_4e_3 = -e_5$
$\mathbf{B}_{05}$	:	$e_1e_2 = e_3 + e_5$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_3 = -e_5$	
$\mathbf{B}_{06}$	:	$e_1e_2 = e_3 + e_5$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_1 = e_5$	$e_4e_3 = -e_5$
$\mathbf{B}_{07}$	:	$e_1e_2 = e_3 + e_5$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_3 = -e_5$	$e_4e_4 = e_5$
$\mathbf{B}_{08}$	:	$e_1e_2 = e_3 + e_5$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_1 = e_5$	$e_4e_3 = -e_5$
$\mathbf{B}_{09}^{\alpha}$	:	$e_1e_2 = e_3 + e_5$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_1 = e_5$	
		$e_4e_2 = e_5$	$e_4e_3 = -e_5$	$e_4e_4 = \alpha e_5$		
$\mathbf{B}_{10}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_3 = -e_5$
$\mathbf{B}_{11}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_3 = -e_5$
$\mathbf{B}_{12}^{\alpha}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	
		$e_4e_1 = e_5$	$e_4e_3 = -e_5$	$e_4e_4 = \alpha e_5$		
$\mathbf{B}_{13}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	$e_4e_2 = e_5$
$\mathbf{B}_{14}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_4 = e_5$	
		$e_4e_2 = e_5$	$e_4e_3 = -e_5$	$e_4e_4 = e_5$		

The algebraic classification of 4-dimensional nilpotent mono Leibniz algebras

Theorem (3)

Up to isomorphism, there are infinitely many complex 4-dimensional nilpotent (non-binary Leibniz) mono Leibniz algebras, described explicitly in terms of 10 one-parameter families and 12 additional isomorphism classes.

$\mathbb{M}_{01}$	:	$e_1e_1 = e_2$	$e_2e_3 = e_4$			
$\mathbb{M}_{02}$	:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_3e_1 = e_4$		
$\mathbb{M}_{03}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_4$	$e_2e_1 = -e_3$	$e_3e_1 = (1-\alpha)e_4$	
$\mathbb{M}_{04}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_4$	$e_2e_1 = -e_3$	$e_2e_2 = e_4$	$e_3e_1 = (1-\alpha)e_4$
$\mathbb{M}_{05}$	:	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_2e_3 = e_4$		
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		$e_2e_3 = e_4$	$e_3e_1 = e_4$	$e_3e_2 = -e_4$		
$\mathbb{M}_{07}$	:	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_3 = e_4$		
$\mathbb{M}_{08}$	:	$e_1e_2 = e_3 + e_4$	$e_2e_1 = -e_3$	$e_3e_3 = e_4$		

$$\mathbb{M}_{09},\ \mathbb{M}_{10}^{\alpha},\ \mathbb{M}_{11}, \ldots\ \mathbb{M}_{19},\ \mathbb{M}_{20},\ \mathbb{M}_{21},\ \mathbb{M}_{22}.$$

Classification of 4-dimensional nilpotent algebras with nil-index 3

An element $x \in A$ is called nilpotent, if there is an integer $r \geq 1$ such that $x^r = 0$. If any element in A is nilpotent, then A is called a nil-algebra. Now A is called a nil-algebra of nil-index $n \geq 2$, if $y^n = 0$ for all $y \in A$ and there is $x \in A$ such that $x^{n-1} \neq 0$.

1. A Lie algebra is a nil-algebra with nil-index 2.
2. A symmetric Leibniz algebra is a nil-algebra with nil-index 3.
3. A dual alternative algebras is a nil-algebra with nil-index 3.
4. Commutative nil-algebras with nil-index 3 are Jordan algebras.
5. Any finite-dimensional Jordan nil-algebra is nilpotent. [8]
6. The intersection of left mono Leibniz and right mono Leibniz algebras gives the variety of nil-algebras of nil-index 3. [9]

[8] Schafer R.D., Academic Press (1966).

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Let $\mathfrak{n}$ be a complex 4-dimensional nilpotent algebra of nil-index 3. Then $\mathfrak{n}$ is a 2-step nilpotent algebra or isomorphic to one algebra from the following list:

$\mathfrak{L}_{01}$:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_1 = -e_3$	$e_3e_1 = -e_4$		
$\mathfrak{L}_{09}$:	$e_1e_2 = -e_3 + e_4$	$e_1e_3 = -e_4$	$e_2e_1 = e_3$	$e_3e_1 = e_4$		
$\mathfrak{L}_{10}$:	$e_1e_2 = -e_3$	$e_1e_3 = -e_4$	$e_2e_1 = e_3$	$e_2e_2 = e_4$	$e_3e_1 = e_4$	
$\mathfrak{L}_{11}$:	$e_1e_1 = e_4$	$e_1e_2 = -e_3$	$e_1e_3 = -e_4$	$e_2e_1 = e_3$	$e_2e_2 = e_4$	$e_3e_1 = e_4$
$\mathfrak{L}_{12}$:	$e_1e_1 = e_4$	$e_1e_2 = -e_3$	$e_1e_3 = -e_4$	$e_2e_1 = e_3$	$e_3e_1 = e_4$	
M_{03}^{α} :	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_4$	$e_2e_1 = -e_3$	$e_3e_1 = (1-\alpha)e_4$		
M_{04}^{α} :	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_4$	$e_2e_1 = -e_3$	$e_2e_2 = e_4$	$e_3e_1 = (1-\alpha)e_4$	
M_{05} :	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_2e_3 = e_4$	$e_3e_1 = e_4$	$e_3e_2 = -e_4$	
M_{06} :	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_2e_2 = e_4$	$e_2e_3 = e_4$	$e_3e_1 = e_4$	$e_3e_2 = -e_4$
M_{07} :	$e_1e_2 = e_3$	$e_2e_1 = -e_3$	$e_3e_3 = e_4$			
M_{08} :	$e_1e_2 = e_3 + e_4$	$e_2e_1 = -e_3$	$e_3e_3 = e_4$			
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Geometric classification

The degenerations between the (finite-dimensional) algebras from a certain variety $\mathfrak{V}$ defined by a set of identities have been actively studied in the past decade. The description of all degenerations allows one to find the so-called rigid algebras and families of algebras, i.e., those whose orbit closures under the action of the general linear group form irreducible components of $\mathfrak{V}$ (with respect to the Zariski topology).

Let T be a set of polynomial identities. The set of algebra structures on $\mathbb{V}$ satisfying polynomial identities from T forms a Zariski-closed subset of the variety $\mathrm{Hom}(\mathbb{V} \otimes \mathbb{V}, \mathbb{V})$. We denote this subset by $\mathbb{L}(T)$. The general linear group $\mathrm{GL}(\mathbb{V})$ acts on $\mathbb{L}(T)$ by conjugations:

$$(g * \mu)(x \otimes y) = g\mu(g^{-1}x \otimes g^{-1}y)$$

for $x, y \in \mathbb{V}$, $\mu \in \mathbb{L}(T) \subset \mathrm{Hom}(\mathbb{V} \otimes \mathbb{V}, \mathbb{V})$ and $g \in \mathrm{GL}(\mathbb{V})$. Thus, $\mathbb{L}(T)$ is decomposed into $\mathrm{GL}(\mathbb{V})$-orbits that correspond to the isomorphism classes of algebras. Let $O(\mu)$ denote the orbit of $\mu \in \mathbb{L}(T)$ under the action of $\mathrm{GL}(\mathbb{V})$ and $\overline{O(\mu)}$ denote the Zariski closure of $O(\mu)$.

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Geometric classification

Let **A** and **B** be two n-dimensional algebras satisfying the identities from T, and let $\mu, \lambda \in \mathbb{L}(T)$ represent **A** and **B**, respectively. We say that **A** degenerates to **B** and write **A** $\to$ **B** if $\lambda \in \overline{O(\mu)}$. Note that in this case we have $\overline{O(\lambda)} \subset \overline{O(\mu)}$.

Let **A** be represented by $\mu \in \mathbb{L}(T)$. Then **A** is *rigid* in $\mathbb{L}(T)$ if $O(\mu)$ is an open subset of $\mathbb{L}(T)$. Recall that a subset of a variety is called irreducible if it cannot be represented as a union of two non-trivial closed subsets. A maximal irreducible closed subset of a variety is called an *irreducible component*.

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In many cases, the irreducible components of the variety are determined by the rigid algebras, i.e. algebras whose orbit closure is an irreducible component. It is worth mentioning that this is not always the case and Flanigan had shown that the variety of 3-dimensional nilpotent associative algebras has an irreducible component which does not contain any rigid algebras. It is, instead, defined by the closure of a union of a one-parameter family of algebras. Kaygorodov, Khrypchenko, Lopes proved (2023) the following theorems

Theorem (5)

For any $n \geq 2$, the variety of all n-dimensional nilpotent algebras is irreducible and has dimension $\frac{n(n-1)(n+1)}{3}$.

Theorem (6)

For any $n \geq 2$, the variety of all n-dimensional commutative nilpotent algebras is irreducible and has dimension $\frac{n(n-1)(n+4)}{6}$.

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For any $n \geq 2$, the variety of all n-dimensional anticommutative nilpotent algebras is irreducible and has dimension $\frac{(n-2)(n^2+2n+3)}{6}$.

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2. Two-dimensional algebras [Kaygorodov, Volkov (2019)]
3. Three-dimensional Novikov algebras [Benes, Burde (2014)]
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The blue parts contain all possible degenerate. In the remaining parts, only open components were found.

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Geometric classification of four dimensional algebras

1. 4-dimensional Lie algebras [Burde, Steinhoff, (1999)]
2. 4-dimensional Leibniz algebras [Ismailov, Kaygorodov, Volkov (2018)]
3. 4-dimensional Zinbiel algebras [Kaygorodov, Popov, Pozhidaev, Volkov (2018)]
4. 4-dimensional nilpotent commutative algebras [Fernandez, Kaygorodov, Khrypchenko, Volkov (2022)]
5. 4-dimensional binary Lie algebras [Kaygorodov, Popov, Volkov, (2018)]
6. 4-dimensional nilpotent Novikov algebras [Karimjanov, Kaygorodov, Khudoyberdiyev (2019)]
7. 4-dimensional nilpotent noncommutative Jordan algebras [Jumaniyozov, Kaygorodov, Khudoyberdiyev (2021)]
8. 4-dimensional nilpotent Poisson algebras [Abdelwahab, Barreiro, Calderon, Ouaridi (2023)]

Geometric classification of $n \geq 5$ dimensional algebras

1. 5-dimensional nilpotent Malcev algebras [Kaygorodov, Popov, Volkov, (2018)]
2. 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras [Jumaniyozov, Kaygorodov, Khudoyberdiyev (2021)]
3. 5-dimensional nilpotent symmetric Leibniz algebras [Alvarez, Kaygorodov (2021)]
4. 5-dimensional nilpotent associative algebras [Ignatyev, Kaygorodov, Popov (2021)]
5. 5-dimensional Zinbiel algebras [Alvarez, Junior, Kaygorodov (2022)]
6. 6-dimensional nilpotent Lie algebras [Grunewald, O'Halloran, (1988)]
7. 6-dimensional nilpotent Tortkara algebras [Gorshkov, Kaygorodov, Khrypchenko (2020)]
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Geometric classification of 5-dimensional nilpotent Leibniz and binar Leibniz algebras

Theorem (8)

The variety of complex 5*-dimensional nilpotent Leibniz algebras has dimension* 24 *and it has* 10 *irreducible components (in particular, there is only one rigid algebra in this variety).*

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The variety of complex 4-dimensional nilpotent mono Leibniz algebras has dimension 15 and it has 3 irreducible components (in particular, there is only one rigid algebra in this variety).

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Asymptotic estimates for components

Neretin Yu.A. proved the following theorems[10].

Theorem (12)

The dimension of any Lie_n component does not exceed $\frac{2}{27}n^3 + O(n^{8/3})$.

Theorem (13)

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Asymptotic estimates for components

Kashuba I., Shestakov I. proved the following theorems[11].

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Let Ω be an arbitrary family of non-isomorphic n-dimensional alternative algebras over an algebraically closed field k. The dimension of any Ω component does not exceed $\frac{4}{27}n^3 + O(n^{8/3})$.

Theorem (16)

Let Ω be an arbitrary family of non-isomorphic n-dimensional Jordan algebras over an algebraically closed field k. The dimension of any Ω component does not exceed $\frac{1}{6\sqrt{3}}n^3 + O(n^{8/3})$.

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