

Lie algebras, their invariants, and applications

Habilitační přednáška

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A terminological note

Sophus Lie
(a person)



Lie algebras,
Lie groups
(objects)

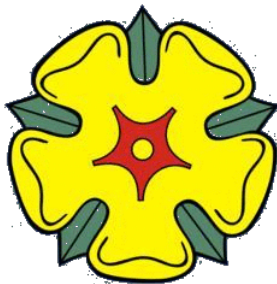
What Lie algebras are good for?

Groups \rightsquigarrow Lie groups \rightsquigarrow Lie algebras

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Groups \rightsquigarrow Lie groups \rightsquigarrow Lie algebras

Groups = symmetries



\mathbb{Z}_5



\mathbb{D}_6

What Lie algebras are good for?

Groups \rightsquigarrow Lie groups \rightsquigarrow Lie algebras

Lie groups = continuous symmetries

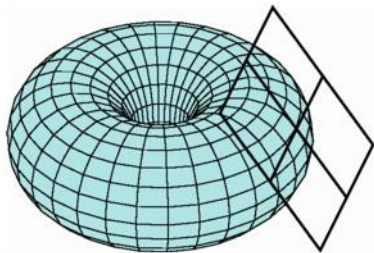
$$\frac{\partial u}{\partial t} - \lambda \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 0$$

$\frac{\partial}{\partial x}$	space translation
$\frac{\partial}{\partial t}$	time translation
$u \frac{\partial}{\partial u}$	scalings
$x \frac{\partial}{\partial x} + 2t \frac{\partial}{\partial t}$	
$2t \frac{\partial}{\partial x} - xu \frac{\partial}{\partial u}$	Galilean transform
$4xt \frac{\partial}{\partial x} + 4t^2 \frac{\partial}{\partial t} - (x^2 + 2t)u \frac{\partial}{\partial u}$	inversion
$\alpha(x, t) \frac{\partial}{\partial u}$	linear transforms

What Lie algebras are good for?

Groups \rightsquigarrow Lie groups \rightsquigarrow Lie algebras

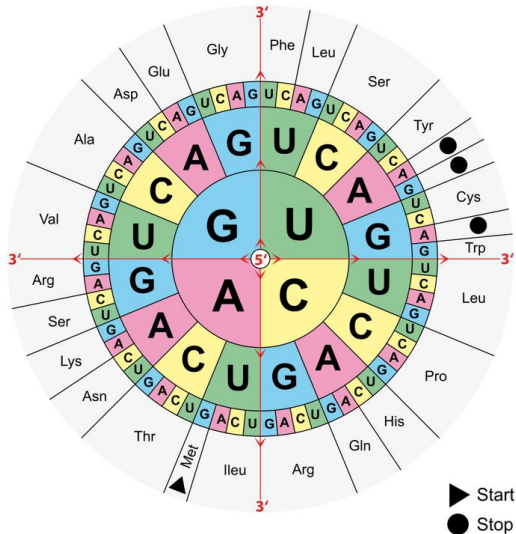
Lie algebras = linearizations of Lie groups



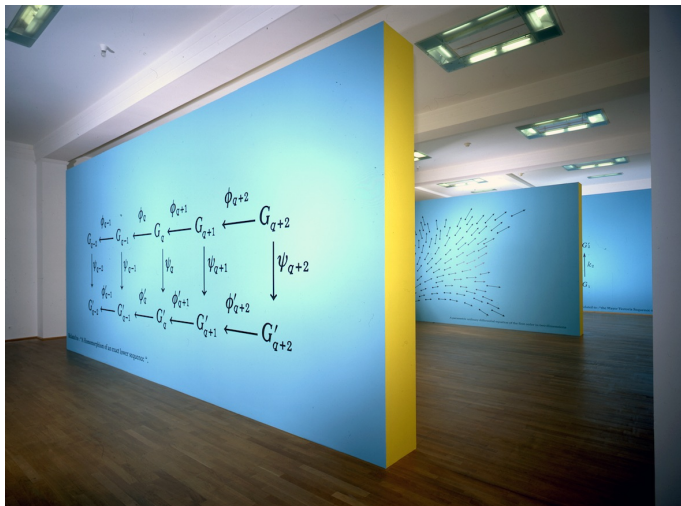
	e	h	f
e	0	$-2e$	h
h	$2e$	0	$-2f$
f	$-h$	$2f$	0

What Lie algebras are good for?

Genetic code \rightsquigarrow representations of Lie algebras

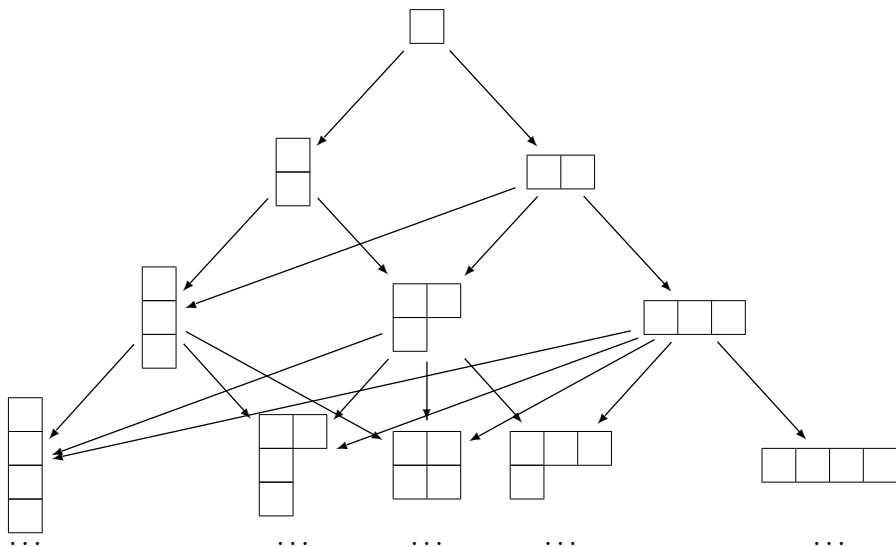


Lie algebras Homological algebra
 \rightsquigarrow Cohomology of Lie algebras



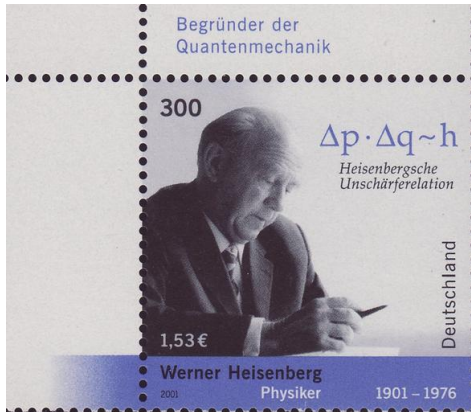
Bernar Venet, Acrylic on wall, 2002

Cohomology of Lie algebras



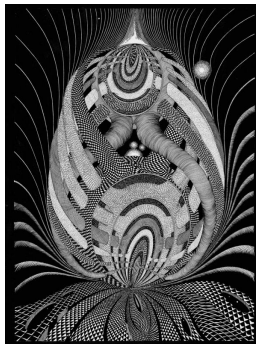
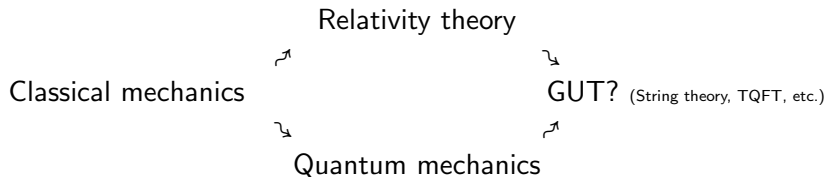
What cohomology of Lie algebras is good for?

Quantum mechanics: uncertainty principle \leadsto 2nd cohomology



What cohomology of Lie algebras is good for?

Deformations \rightsquigarrow 2nd and 3rd cohomology

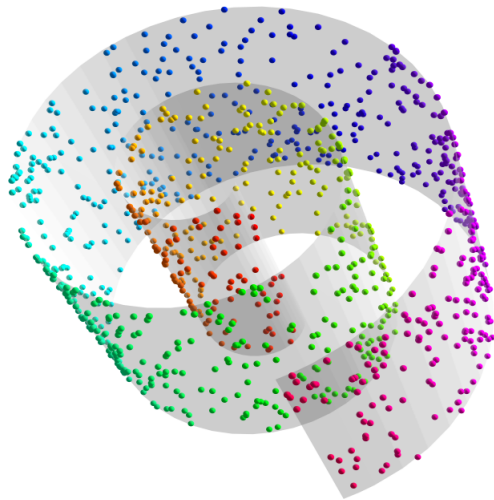


Anatoly Fomenko, Deformation of the Riemann Surface of an Algebraic Function, India ink and pencil on paper, 44 x 62 cm, 1983

What else homological algebra is good for?

Clustering

Number of clusters = 0th cohomology



What else?

My other interests:

- ▶ History
- ▶ Mathematical Logic
- ▶ Combinatorics
- ▶ Analysis (divergent series)
- ▶ Symbolic computations
- ▶ Statistics
- ▶ Applications of mathematics to biology

Another way of finding the constant is as follows - 41

Let us take the series $1+2+3+4+5+\dots$. Let c be its constant. Then $c = 1+2+3+4+\dots$

$$\therefore 4c = 4 + 8 + \dots$$

$$\therefore -3c = 1-2+3-4+\dots = \frac{1}{(1+1)^2} = \frac{1}{4}$$

$$\therefore c = -\frac{1}{12}$$

To je vše. Děkuji.