

Special and exceptional mock-Lie algebras (extended abstract)

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Mock-Lie algebras are algebras satisfying two identities: commutativity

$$xy = yx$$

and the Jacobi identity

$$(xy)z + (zx)y + (yz)x = 0.$$

It is almost immediate that such algebras are Jordan algebras of nil index 3 (i.e., $x^3 = 0$). Conversely, over a field of characteristic $\neq 2, 3$, any Jordan algebra of nil index 3 is mock-Lie. Such algebras appeared in the literature under different names: Lie-Jordan, Jacobi-Jordan, “commutative” Lie algebras, etc. They live a dual life: as a very particular class of Jordan algebras, and as strange cousins of Lie algebras, and this work is an interesting blend of Lie and Jordan theories (and, of course, computer algebra).

The main question we are concerned with is which of those algebras admit a faithful representation, or, what is the same, admit embedding into an associative algebra (i.e., in Lie parlance, satisfy the Ado theorem, or, in Jordan parlance, are special).

The arguments used to establish the Ado theorem and the Poincaré–Birkhoff–Witt theorem – a fact closely related with the possibility of embedding of Lie algebras into associative ones – fail in somewhat curious ways in the mock-Lie case (one of those ways involves calculation of Gröbner bases of universal enveloping algebras of mock-Lie algebras).

Mock-Lie algebras of low dimension (≤ 6 at least) are special (i.e., embedded into an associative algebra). On the other hand, an exceptional (i.e., not special) mock-Lie algebra was constructed a long time ago in an unpublished preprint [1], and we reproduce these old efforts, showing how for any Jordan s -identity (Glennie, Thedy, Medvedev, etc.) one may produce a mock-Lie algebra which does not satisfy this identity, and hence is exceptional. The minimal dimension of so constructed algebra is 44.

Substantial computer calculations are involved, utilizing Albert [3] and GAP.

There are many open questions related to mock-Lie algebras: about minimal dimension and minimal degree of nilpotency of exceptional algebras, about cohomology theory, about mock-Lie and dual to it operads, etc.

Based on [2].

References

- [1] I.R. Hentzel, D.P. Jacobs, and S.R. Sverchkov, *On exceptional nil of index 3 Jordan algebras*, Preprint, Novosibirsk State Univ., 1997.
- [2] P. Zusmanovich, *Special and exceptional mock-Lie algebras*, *Lin. Algebra Appl.* **518** (2017), 79–96; arXiv:1608.05861.
- [3] Albert; <http://www1.osu.cz/~zusmanovich/albert/>

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