A small step in classification of simple Lie algebras in characteristic 2

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Classification problem of simple Lie algebras

- $p = 0$: classic (Killing, E. Cartan, ...)
- $p > 3$: a 3-volume set by H. Strade (the last one appeared in 2012)
- $p = 2, 3$: open

Toral rank

$\boxed{p = 2}$

First surprise

The 3-dimensional simple Lie algebra S

$$[e, h] = e, \quad [f, h] = f, \quad [e, f] = h$$

has absolute toral rank 2. Toral elements: $h + e + e^{[2]}$, $h + f + f^{[2]}$.

Theorem (Skryabin 1998)

There are no finite-dimensional simple Lie algebras over an algebraically closed field of absolute toral rank 1.

Ongoing project (Grishkov & Premet)

Absolute toral rank 2?

Toral rank (cont.)

$p = 2$

Theorem (Grishkov & Zusmanovich, 2014)

A finite-dimensional simple Lie algebra over an algebraically closed field, of absolute toral rank 2, and having a Cartan subalgebra of toral rank 1, is isomorphic to S.

Next goal

Simple Lie algebras having a Cartan subalgebra of toral rank 1.

Ingredients of the proof

$p = 2$

- Description of simple Lie algebras having a Cartan subalgebra of toral rank 1 as certain filtered deformations (Skryabin 1998).
- Computation of these filtered deformations in some cases.
- Low-degree cohomology of $S\otimes$ (commutative associative algebra).
- Dealing with a certain family of 15-dimensional simple Lie algebras.
- GAP.

A family of 15-dimensional algebras

$\boxed{p = 2}$

A 2-parameter deformation of

$$S \otimes \mathcal{O}_1(2) + f^{[2]} \otimes \langle 1, x \rangle + \partial$$

(only those products are listed which differ in the deformed algebra).

Some properties:

- simple;
- of absolute toral rank 3;
- $\mathrm{H}^2(L, K) = 0$;
- do not possess symmetric invariant bilinear forms;
- p-envelope coincides with derivation algebra and is of dimension 19;
- subalgebra generated by absolute zero divisors is of dimension 7.

A family of 15-dimensional algebras

$p = 2$

$$\{e \otimes 1, \quad e \otimes x\} = f^{[2]} \otimes \alpha x$$

$$\{e \otimes 1, \quad e \otimes x^{(2)}\} = f^{[2]} \otimes \beta 1$$

$$\{e \otimes 1, \quad e \otimes x^{(3)}\} = \{e \otimes x, \quad e \otimes x^{(2)}\} = f^{[2]} \otimes \beta x + \partial$$

$$\{e \otimes x, \quad e \otimes x^{(3)}\} = h \otimes 1$$

$$\{e \otimes x^{(2)}, e \otimes x^{(3)}\} = h \otimes x$$

$$\{e \otimes x, \quad h \otimes x^{(3)}\} = f \otimes 1$$

$$\{e \otimes x^{(2)}, h \otimes x^{(3)}\} = f \otimes x$$

$$\{e \otimes x^{(3)}, h \otimes x^{(3)}\} = f \otimes x^{(2)}$$

$$\{e \otimes x, \quad f \otimes x^{(3)}\} = \{e \otimes x^{(3)}, f \otimes x\} = f^{[2]} \otimes 1$$

$$\{e \otimes x^{(2)}, f \otimes x^{(3)}\} = \{e \otimes x^{(3)}, f \otimes x^{(2)}\} = f^{[2]} \otimes x$$

$$\{h \otimes x, \quad h \otimes x^{(3)}\} = f^{[2]} \otimes 1$$

$$\{h \otimes x^{(2)}, h \otimes x^{(3)}\} = f^{[2]} \otimes x$$

$$\{e \otimes 1, \quad \partial\} = f \otimes \alpha x^{(3)}$$

“Commutative” cohomology

$p = 2$

“Commutative” Lie algebras:

$$[x, y] = [y, x] \quad \text{(instead of } [x, x] = 0\text{)}$$

$+$ Jacobi identity.

A natural cohomology in this class: in the Chevalley–Eilenberg complex, replace alternating cochains by symmetric ones.

Question

Derived functor? Universal enveloping algebra?

That's all. Thank you.

Slides at `http://justpasha.org/math/porto.pdf`