EXAM FOR "MEASURE THEORY AND INTEGRATION" (TMAIN)
WINTER SEMESTER 2018/2019

QUESTIONS SET No. 1

1. Give the definition of a ring of sets. With respect to which operations any ring of sets forms an (algebraic) ring? Prove it.

Is it possible to represent:

 (i) the ring $\mathbb{Q}$ (rational numbers);
 (ii) the ring $GF(2)$ (field of two elements);
 (iii) the ring $M_2(GF(2))$ (2×2 matrices over the field of two elements)

in such a form?

2. Prove that for any measurable non-negative function f, and any real number $a > 0$,

$$\int f^a \, \mathrm{d}\mu = a \int_0^\infty t^{a-1} \mu(f > t) \, \mathrm{d}t.$$