EXAM FOR "MEASURE THEORY AND INTEGRATION" (TMAIN)
WINTER SEMESTER 2018/2019

QUESTIONS SET NO. 2

1. Prove the existence of Borel non-measurable sets in $\mathbb{R}^n$.

2. Let μ be a measure on a set S. Prove that there exist measurable sets $A_1 \subset A_2 \subset \dots$ such that $\bigcup_{n\geq 1} A_n = S$ and $\mu(A_n) < \infty$ for any n, if and only if there exists a measurable positive (i.e., $f(x) > 0$ for any $x \in S$) function f such that $\int f \,\mathrm{d}\mu < \infty$.