EXAM FOR "MEASURE THEORY AND INTEGRATION" (TMAIN)
WINTER SEMESTER 2018/2019

QUESTIONS SET NO. 3

1. Formulate the theorem about connection between the Lebesgue and Riemann integrals. Why this theorem is important?

Give an example of function which is Lebesgue integrable but not Riemann integrable.

2. Prove that if $f : A \to \mathbb{R} \cup \{+\infty\}$ is an integrable function such that $a \leq f(x) \leq b$ for any $x \in A$ and some fixed $a, b \in \mathbb{R} \cup \{+\infty\}$, then

$$a\mu(A) \leq \int_A f \,\mathrm{d}\mu \leq b\mu(A).$$