

**EXAM FOR “ALGEBRAIC STRUCTURES” (6ALGS)  
SUMMER SEMESTER 2023/2024**

QUESTIONS SET NO. 2

**1.**

**1.1.** Give definition of the order of a group, and the order of an element of a group. Illustrate these notions by examples.

**1.2.** Formulate and prove the Lagrange theorem about subgroups of a group. Give examples illustrating the theorem.

**1.3.** Formulate and prove some consequences of the Lagrange theorem. (Hint: order of an element, cyclic groups).

**2.**

Does there exist a cyclic group which is isomorphic to the direct product of two nontrivial groups?

**3.**

Let  $M_2(\mathbb{Q})$  be the ring of  $2 \times 2$  matrices with rational coefficients,  $R = \left\{ \begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} \mid a, b \in \mathbb{Q} \right\}$ , and  $I = \{A \in R \mid A^2 = 0\}$ .

- (i) Prove that  $R$  is a noncommutative subring of  $M_2(\mathbb{Q})$ .
- (ii) Prove that  $I$  is an ideal in  $R$ .
- (iii) Prove that  $R/I \simeq \mathbb{Q}$ .