Th. Any ring consisting of two elements is isomorphic to one of two rings: $GF(2)$ and square-zero ring.

$\parallel$

the ring with trivial multiplication

$\langle R, + \rangle$ — the additive group of the ring R.

$x^2 \overset{df}{=} x \cdot x = 0$

$x^n = \underbrace{x \cdot \ldots \cdot x}_{n}$ $n>0$

nx

$-nx = \underbrace{(-x) + \ldots + (-x)}_{n}$

$= -(nx)$

$R^2 = 0$

$\langle xy \mid x, y \in R \rangle$

$\parallel df$

$\{x_1 y_1 + \ldots + x_k y_k \mid x_i, y_i \in R\}$

G

$A, B \subseteq G$

$AB \overset{df}{=} \{ab \mid a \in A, b \in B\}$

Square-zero rings.

$\langle R, +, \cdot \rangle$

$\langle R, + \rangle$

$R^2 = 0 \iff xy = 0 \ \forall_{x,y \in R}$

$\langle \mathbb{Q}, + \rangle$

$\langle \mathbb{R}, + \rangle$