

$$R_1 \oplus \dots \oplus R_n \quad \{(\ast, \dots, \ast) \mid r_i \in R_i\}$$

$$(R_1 \oplus R_2) \oplus R_3 \cong R_1 \oplus (R_2 \oplus R_3)$$

$$R_1 \oplus R_2 \quad 0, R_1 \oplus R_2$$

$$R_1, R_2$$


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Def. A ring  $R$  is called commutative if  $ab = ba \quad \forall a, b \in R$

$$(R[x], +, \cdot) \quad (a_0, a_1, a_2, \dots, a_n, \dots)$$

$$\begin{aligned}
 & (a_0 + a_1x + a_2x^2 + \dots + a_nx^n) + (b_0 + b_1x + b_2x^2 + \dots + b_nx^n) \\
 &= (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + \dots + (a_n + b_n)x^n
 \end{aligned}$$

$$R[x] \cong R \oplus R \oplus \dots \oplus R \oplus \dots$$

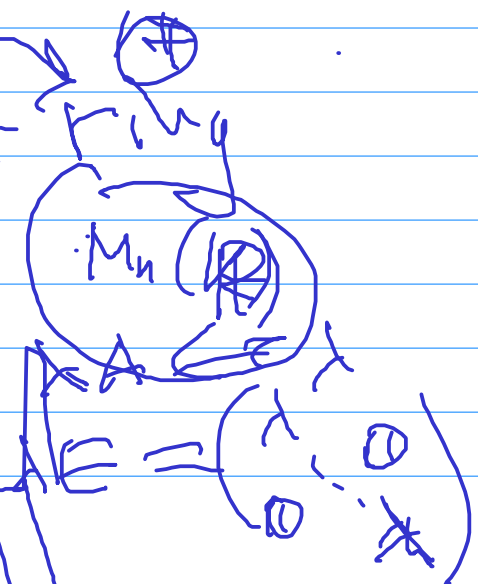
$$(a_0 + a_1x + \dots + a_nx^n) \cdot (b_0 + b_1x + \dots + b_mx^m) =$$

$\mathbb{R}$   
 $a_0 \leftrightarrow a_{00}$

**Algebra** = vector space + ring

$$\lambda(f+g) = \lambda f + \lambda g$$

$$\lambda(fg) = (\lambda f)g$$



$$x^n \cdot x^m = x^{n+m} \quad \mathbb{R}[x]$$

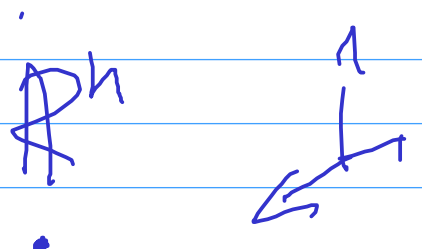
$\langle A, \oplus, \otimes, \cdot \rangle$ , dek

~~$\langle A, \oplus, \otimes, \cdot \rangle$~~

$+$ :  $A \times A \rightarrow A$

$\cdot$ :  $A \times A \rightarrow A$

$\cdot$ :  $K \times A \rightarrow A$



Def. Algebra  $\langle A, K, +, \cdot, * \rangle$

such that  $\langle A, +, \cdot \rangle$  is a ring  
and  $\langle A, K, +, * \rangle$  is a vector space

and  $(\lambda \mu) a = \lambda(\mu a)$   
 $\forall \lambda, \mu \in K \forall a, b \in A$

$\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} = \langle a + bi + ck \rangle$

$\{a + bi \mid a, b \in \mathbb{R}\} = \langle 1, i \rangle$

$i^2 = -1$

$j, k$

$\sqrt{-1}$  and  $i^2 = -1$

$(a + bi) \cdot (c + di) = ac + bci + adi + bdi$

$= (ac - bd) + (bc + ad)i$

$i^2 = -1$   
 $k^2 = -1$   
 $ik = -ki$