$R_1 \oplus \ldots \oplus R_n \quad \{(r_1, \ldots r_n) \mid r_i \in R_i\}$

$(R_1 \oplus R_2) \oplus R_3 \simeq R_1 \oplus (R_2 \oplus R_3)$

$R_1 \oplus R_2 \qquad 0, R_1 \oplus R_2$

R_1, R_2

Def. A ring R is called commutative if $ab = ba \ \forall a, b \in R$

$(R[x], +, \cdot) \qquad (a_0, a_1, a_2, \ldots a_n, \ldots)$

$a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \quad 0 \ldots$

$+ b_0 + b_1 x + b_2 x^2 + \ldots + \ldots + b_m x^m$

$R[x] \simeq R \oplus R \oplus \ldots \oplus R \oplus \ldots$

$(a_0 x^0 + a_1 x + \ldots + a_n x^n) \cdot \qquad R$

$\cdot (b_0 + b_1 x + \ldots + b_m x^m) = \qquad a_0 \leftrightarrow a_0 x^0$

$\boxed{\text{Algebra}}$ = vector space + ring

$\lambda(f+g) = \lambda f + \lambda g$

$\lambda \cdot (fg) = (\lambda \cdot f) \cdot g$

$M_n(\mathbb{R})$

$\lambda E = \begin{pmatrix} \lambda & & 0 \\ & \ddots & \\ 0 & & \lambda \end{pmatrix}$

$\boxed{x^n \cdot x^m = x^{n+m}}$

$\mathbb{R}[x] \quad f(x) \cdot \mathbb{R}[x]$

$\langle A, \oplus, \otimes, \cdot_\lambda \rangle, \lambda \in K$

$+ : A \times A \to A$

$\cdot : A \times A \to A$

$\boxed{\cdot_\lambda : K \times A \to A}$

$\mathbb{R}^n$

<u>Def.</u> Algebra $\langle A, K, +, \cdot, * \rangle$

such that $\langle A, +, \cdot \rangle$ is a ring

and $\boxed{\langle A, K, +, * \rangle \text{ is a vector space}}$

and $(\lambda * a) b = \lambda * (ab)$

$\forall \lambda \in K \; \forall a, b \in A$

$\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} = \langle 1, i, j, k \rangle$

$\{a + bi \mid a, b \in \mathbb{R}\} = \langle 1, i \rangle$

j, k $\qquad \boxed{i^2 \stackrel{df}{=} -1}$ $\qquad j^2 = -1$

$k^2 = -1$

$\sqrt{-1}$ $\qquad 1 \cdot i = i \cdot 1 = i$ $\qquad \langle 1, j \rangle \cong \mathbb{C}$

$\langle 1, k \rangle \cong \mathbb{C}$

$(a + bi) \cdot (c + di) = ac + bi \cdot c + a \cdot di + bi \cdot di$

$= (ac - bd) + (bc + ad) i + \cancel{bd\, i^2}$