EXAM FOR "ALGEBRAIC STRUCTURES" (6ALGS)
SUMMER SEMESTER 2025/2026

QUESTIONS SET NO. 1

1.
1.1. Define normal subgroup of a group. Is each subgroup normal?
1.2. Define group homomorphism, and its kernel and image. Illustrate these notions by examples.
1.3. What is the relationship between the kernel of a group homomorphism, and a normal subgroup? Formulate the corresponding theorem and prove it.

2.
Find two finite groups of the same order that are not isomorphic. Show that they are not isomorphic. What is the smallest number n such that there exist two non-isomorphic groups of order n?

3.
Recall that the center $Z(R)$ of a ring R is defined as the set of all elements $z \in R$ which commute with any other element $x \in R$: $xz = zx$. Prove that if $x^2 - x \in Z(R)$ holds for every $x \in R$, then R is commutative.