EXAM FOR "ALGEBRAIC STRUCTURES" (6ALGS)
SUMMER SEMESTER 2025/2026

QUESTIONS SET NO. 2

1.
1.1. Give definition of the order of a group, and the order of an element of a group. Illustrate these notions by examples.
1.2. Formulate and prove the Lagrange theorem about subgroups of a group. Give examples illustrating the theorem.
1.3. Formulate and prove some consequences of the Lagrange theorem. (Hint: order of an element, cyclic groups).

2.
Does there exist a cyclic group which is isomorphic to the direct product of two nontrivial groups?

3.
Let $M_2(\mathbb{Q})$ be the ring of 2×2 matrices with rational coefficients, $R = \left\{ \begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} \mid a,b \in \mathbb{Q} \right\}$, and $I = \{A \in R \mid A^2 = 0\}$.

(i) Prove that R is a noncommutative subring of $M_2(\mathbb{Q})$.
(ii) Prove that I is an ideal in R.
(iii) Prove that $R/I \simeq \mathbb{Q}$.