

**EXAM FOR “ALGEBRAIC STRUCTURES” (6ALGS)
SUMMER SEMESTER 2025/2026**

QUESTIONS SET NO. 3

1.

Formulate and prove the second and third homomorphism theorems for groups.

2.

Prove that the eight matrices

$$\pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \pm \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \pm \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \pm \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

form a subgroup of $GL_2(\mathbb{C})$. List as much properties of this group as you can.

3.

Explicitly construct the addition and multiplication tables for the field consisting of:

- 1) 3 elements;
- 2) 4 elements;
- 3) 6 elements.

Describe all instances when one of these fields is a subfield of the other.