3020 DIFFERENTIAL EQUATION

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Each lecture lasted 1 h 15 min. The textbook is P. Blanchard, R.L. Devaney, G.R. Hall, *Differential Equations*, 4th ed., Brooks/Cole.

Lecture 1

According to §1.1 of the texbook.

What differential equations are and why are they important.

Simplest differential equations $y' = 0$ and $y'' = 0$ and their solutions. First- and higher-order differential equations

Population growth: $\frac{dP}{dt} = kP$.

Methods of solutions:

(i) guessing – take $P(t) = ae^{bt} + c$;
(ii) power series.

Initial condition $P(0) = P_0$, initial-value problem.

Logistic equation: $\frac{dP}{dt} = k(1 - \frac{P}{N})P$. Equilibrium solutions: $P = 0$ and $P = N$.

Predator-prey model: foxes and rabbits

$$\frac{dR}{dt} = \alpha R - \beta RF$$
$$\frac{dF}{dt} = -\gamma F + \delta RF,$$

where α is growth rate of rabbits, β is a rate with which rabits are eaten by foxes, γ is a death rate of foxes, δ is a rate with which foxes eat rabits.

Systems of differential equations.

Lecture 2

Qualitative picture using logistic equation: $\frac{dP}{dt} = 0.4P(1 - \frac{P}{230})$.

Integration as a method for solving differential equations of the form $\frac{dy}{dx} = f(x)$.

Generalization of this: separation of variables, separable differential equations: $\frac{dy}{dx} = f(x)g(y)$.

Caveat: when we divide by y, we may miss a solution $y = 0$, so we should check this possibility separately.

Lecture 3

Example of power series method: $y' = 2x - 1$.

Example of an initial-value problem: $y' = -xy; y(0) = \frac{1}{\sqrt{\pi}}$, another example from the textbook.

Octave (as alternative to Matlab).

Slope fields.

Lecture 4

Playing with Octave to draw slope fields.

Euler method for numeric solutions of initial value problems. Example: $y' = x; y(0) = 0$.

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Lecture 5

Existence and uniqueness theorems for an initial-value problem (Cauchy-Kowalevskaya theorem(s)). Why they are useful.

Autonomous equations. Examples from the textbook: $y' = 1 + y^2$ and $y' = 3y^{\frac{2}{3}}$, both with $y(0) = 0$.

Equilibrium points, phase lines. 3 types of equilibrium points: sink, source, node. Example from the textbook: $y' = (y - 2)(y + 1)$.

Lecture 6

(According to §1.8) Linear equations. Homogeneous linear equations.

Finding a solution of nonhomegeneous equation by guessing: $\frac{dy}{dx} = -2y + e^x$.

(According to §1.9) Integrating factors.

Lecture 7

Review before the test: qualitative techniques (equilibrium points, phase lines, slope fields), analytic techniques (guessing, separation of variables, integrating factor).

Lecture 8

Mid-term test 1.

Lecture 9

(§§2.1, 2.2 of the textbook).

System of first-order differential equations. Rabbits and foxes again.

Equilibrium solutions. More generally, what happens if just one of the functions, R and F, is constant? (Answer: then it is necessary constant zero, and another one is an exponential function).

Initial conditions.

Two ways graphically represent the solutions: draw the two graphs together, or draw a phase portrait.

Phase plane, solution curve, equilibrium points on the phase portrait.

Symbolic representation of a system: $\frac{dY}{dt} = F(Y)$ for $Y = (x(t), y(t))^t$.

Direction field.

Direction field and phase portrait for the system $\frac{dx}{dt} = x$, $\frac{dy}{dt} = y$ (they are straight lines).

Lecture 10

(§§2.2, 2.3, 2.4 of the textbook).

Equilibrium points, phase portrait of the system

$$\begin{cases} \frac{dx}{dt} = 3x + y \\ \frac{dy}{dt} = x - y \end{cases}$$

Second order differential equation reduced to a system of first-order ones: Harmonic oscillator, $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$, the system

$$\begin{cases} \frac{dx}{dt} = v \\ \frac{dv}{dt} = -\frac{k}{m}x. \end{cases}$$

Phase portrait. Guessing solutions in the case $\frac{k}{m} = 1$: $x = sin(t)$, $v = cos(t)$. Solution curves are circles.

Damped harmonic oscillator leads to a differential equation of the form $\frac{d^2x}{dt^2} + p\frac{dx}{dt} + qx = 0$. Guessing solutions in the form $x(t) = e^{\lambda t}$ leads to a quadratic equation in λ.

Decoupled systems, and partially decoupled systems. An example:

$$\begin{cases} \frac{dx}{dt} = 2x + 3y \\ \frac{dy}{dt} = -4y. \end{cases}$$

Lecture 11

(§2.5)
Euler's method for the systems.

Lecture 12

(§3.1)
Linear systems. "Linearity principle" (the set of solutions forms a vector space).

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