SYLLABUS - 5020 MODERN ALGEBRA, FALL 2013

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This is a course on Lie algebras, with emphasis on computational aspects.

The textbooks are:

(1) N. Jacobson, *Lie Algebras*, Dover, 1979 (reprint of Interscience Publ., 1962).
(2) J.E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer, 1972.
(3) V.S. Varadarajan, *Lie Groups, Lie Algebras, and Their Representations*, Springer, 1984.

We will try to cover the following topics:

(1) Notion of a Lie algebra, place of Lie algebras in mathematics.
(2) Structure constants, multiplication table of a Lie algebra.
(3) Subalgebras, ideals, quotients, direct sums; solvable, nilpotent, simple, semisimple Lie algebras.
(4) Relationship between Lie and associative algebras.
(5) Derivations and automorphisms.
(6) Classification of Lie algebras of low dimension.
(7) Some important Lie algebras (Heisenberg, $sl(2)$, Witt) and their properties.
(8) Rudiments of the structure theory: radical, Levi-Malcev decomposition, root system.
(9) Rudiments of the representation and cohomology theories of Lie algebras.
(10) Computer algebra system GAP.

Students will be expected to complete small projects by computing various invariants of Lie algebras using GAP.

There will be a few homeworks during the course, and a final exam (paper and pencil).

Student learning outcomes:

(1) Have a solid knowledge about basics of Lie algebras.
(2) Be acquainted with some remarkable classes of Lie algebras (matrix algebras, simple finite-dimensional Lie algebras over a field of characteristic zero, Heisenberg algebras, Witt algebra), and be able to identify them as they occur in other branches of mathematics (e.g., as symmetries of some system, or as derivation algebras).
(3) Be able to compute invariants of a given Lie algebra, such as center, derived subalgebra, derivations, automorphisms.
(4) Be able to inspect properties of a given Lie algebra using GAP.
(5) Be able to write simple GAP programs.

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